

Modeling the Effect of Section Overlap on Structural Performance of Portable Telescopic Accessibility Ramps: A Parametric Beam Analysis

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Background/Objective: Portable telescopic ramps give wheelchair users access to steps and thresholds where a permanent ramp cannot be built. Their nested sections connect only through sliding overlaps, yet ramp research concentrates on slope and telescopic-structure research on crane booms. The overlap length of an accessibility ramp has not been examined as a structural design variable. This study asked at what overlap length the mid-span deflection of a portable telescopic ramp exceeds the serviceability limit, and how the number of telescoping sections changes that threshold.

Methods: A 1.228 m aluminium 6061-T6 ramp rail carrying a 699 N mid-span load was modelled with Euler-Bernoulli beam elements assembled by the direct stiffness method in Python. Each overlap was represented as a rotational spring whose stiffness follows a force-couple model, $k = \frac{1}{2} k L^2$. The model reproduced the closed-form beam solution with 0.0000% error, and overlap was then swept from 2 mm to 150 mm across two-, three-, and four-section layouts, 320 cases.

Results: At a 2 mm overlap the mid-span deflection reached 240 mm to 360 mm, against 1.74 mm for a continuous beam. The critical overlaps at the L/360 limit of 3.41 mm were 23.9 mm (two-section), 22.5 mm (three-section), and 29.3 mm (four-section). Maximum bending stress stayed at 14.3 MPa in every case, a safety factor of 16.8.

Conclusions: Serviceability, not strength, governs telescopic ramp design; joint position matters as much as joint count; and the 60 mm design overlap of the reference ramp carries roughly a two-fold margin.

Keywords: telescopic ramp, wheelchair accessibility, section overlap, semi-rigid connection, Euler-Bernoulli beam, direct stiffness method, serviceability limit

Introduction

A wheelchair user faces a step the way a car faces a wall. Ramps remove that wall, and accessibility codes therefore regulate them closely: Indian standard IS 7480 fixes slope, width, and edge protection for ramps serving disabled persons¹, and the national harmonised guidelines carry the same requirements into public buildings². Research attention has followed the codes. Studies of ramp usability concentrate on slope, the variable the codes regulate, and have measured how steeper ramps raise the effort and risk of wheelchair ascent³. A permanent ramp, however, cannot be built at every doorway. Portable ramps fill the gap, and among them the telescopic ramp is the most compact: several nested sections slide together for storage and pull apart for use.

A telescopic ramp is not a rigid beam. Where two sections meet, the outer section, with the larger cross-section, hugs the inner section over an engagement length called the overlap.

When a load steps on the ramp, the ramp sags, and that sag is called deflection. How much it sags depends on how well each overlap carries bending from one section into the next. A long overlap makes the ramp behave like one continuous beam. A short overlap lets the sections pivot almost independently, like a loose door hinge, and the ramp folds into a sharp V at the joint.

Structural engineering has a name for a joint that carries some but not all of the bending moment: a semi-rigid connection. A large body of work models such connections in steel frames as rotational springs, beginning with polynomial moment-rotation fits⁴ and continuing through databases and three-parameter models of connection behaviour^{5,6}. Classification systems place every joint on a scale from pinned to rigid by its rotational stiffness⁷, and Eurocode 3 formalises that scale in design practice⁸. Frame analyses show that ignoring the semi-rigid behaviour of real joints misestimates deflections⁹. On the telescopic side, the published work concentrates on hydraulic crane booms, where the contact stresses between nested segments¹⁰ and the strength of telescopic jibs¹¹

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have been studied with detailed finite element models, and more recent work continues in the same vein, analysing the instability load of a telescopic crane boom built from corotational nested beam elements¹². Portable wheelchair ramps themselves have drawn separate, practical attention that stops short of this question: a comparative evaluation of twelve commercial portable ramps catalogued their design features and performance without analysing an internal joint¹³, and a recent finite element stress simulation of a wheelchair ramp modelled it as a single continuous structure rather than a telescoping assembly¹⁴. Between these two literatures sits an open question. The overlap of a telescopic accessibility ramp is mechanically a semi-rigid connection, but no published study treats its length as the design variable and asks how short it may become before the ramp stops being usable.

The question matters because overlap length is the real design knob of a telescopic ramp. A hinge maker can quote a rotational stiffness for a hinge, but a telescopic ramp has no hinge; the stiffness of each joint is determined by the length of overlap. A manufacturer can read an overlap length off a drawing, understand it, and execute it immediately. A minimum overlap is therefore a specification a designer can actually enforce, in a way that an abstract joint stiffness is not.

This study asked two specific questions. First, at what overlap length does the mid-span deflection of a portable telescopic ramp under its rated load exceed the $L/360$ serviceability limit? Second, does adding more telescoping sections always raise that critical overlap? The working hypothesis was that more sections always means worse performance, since each extra joint adds rotational give.

The scope is deliberately narrow. One independent variable, the section overlap length, is swept while material, cross-section, span, and load stay fixed. The model is linear-elastic beam theory with a static load held at mid-span, the worst case for deflection because the middle is the furthest point from both supports at once in a continuous beam; for a beam with discrete semi-rigid joints this is not automatic, since a load near a soft joint could in principle rotate that joint harder than a mid-span load does, and a dedicated load-position sweep reported in Results confirms mid-span remains the governing position across the range in which this paper's thresholds are read. Contact friction, local wall crushing, and plate effects are excluded by design, so that any change in the answer can be traced to the overlap alone.

The theoretical framework is classical. The ramp rail is an Euler-Bernoulli beam¹⁵, each overlap is a rotational spring in the semi-rigid tradition¹⁶, and the spring stiffness follows from a force-couple model of the two bearing contacts at the ends of the engagement length. The beam is solved by the direct stiffness method, the model is first verified against the closed-form solution, and the overlap is then swept across four ramp layouts. The sections that follow give the full method,

the results of the 320-case sweep, and the design implications.

Methods

Research design

This is a parametric computational study built on a verify-then-sweep design: the numerical model must first reproduce a known closed-form answer before any of its parametric output is trusted, following standard verification practice in computational mechanics^{17,18}. All code was written from scratch in Python 3 using NumPy 1.26 for matrix assembly and solution¹⁹, pandas 2.1 for the results table, and Matplotlib 3.8 for figures²⁰. No commercial finite element package was used, so every stiffness term in the model is inspectable; the code and the full 320-case results table are available from the corresponding author upon reasonable request.

Geometry, material, and load

The geometry comes from the author's RampUp v9 concept design, an unpublished personal design study by the author, not a commercial or third-party product, for a portable telescopic wheelchair ramp: two parallel rails, each built from four nested aluminium U-channel sections, extending to $L = 1.228$ m, rated for a 95 kg user, with a designed section overlap of 60 mm. The material is aluminium alloy 6061-T6, from the 6xxx family widely used for light transport structures²¹, whose extruded structural members are reviewed at length elsewhere²², with Young's modulus $E = 69$ GPa and yield stress 240 MPa. The rated load is $95 \text{ kg} \times g = 9.81 \text{ m/s}^2$, multiplied by a 1.5 dynamic factor to cover bouncing and uneven wheel loading — a round-number engineering margin, since no code specifies a dynamic factor for hand-portable ramps; it is broadly consistent with dynamic amplification factors of roughly 0.2 to 0.5 measured on other human- and vehicle-loaded structures²³ and sits within the impact-and-fatigue testing philosophy of ISO 7176-8 for wheelchairs²⁴, though neither source specifies this exact load case and a direct code basis for 1.5 does not exist, and divided across the two rails, giving $P = 699$ N per rail, applied as a point load at mid-span. Each rail is modelled over the full span as an effective solid rectangular section 100 mm wide and 30 mm deep. The second moment of area of a rectangle is $I = bh^3/12$, which gives $I = 2.25 \times 10^{-7} \text{ m}^4$; depth enters cubed, so the depth does most of the work in resisting bending — precisely, both deflection and bending stress scale with $1/I$, and because $I \propto h^3$ for a rectangular section, depth is the dominant geometric parameter for both. The real v9 rail is a thinner 100 mm by 25 mm channel that stays stiff in service only because its free spans between overlaps are short (Figure 6 draws the two sections to the same scale). The effective 30 mm depth was fixed once,

before the sweep, so that the rigid baseline satisfies the serviceability limit and the overlap remains the only variable under test. This is a deliberate modelling choice, not a measurement of the real channel: every deflection and critical-overlap number in this paper is therefore a property of this idealised surrogate section, not of the physical 100×25 mm channel, whose true second moment of area (open, thin-walled, and in the real part perforated) was not computed. “Connection to the design” in the Discussion is reframed accordingly.

Beam theory and limits

The rail is treated as a simply supported Euler-Bernoulli beam, resting on the ground at one end and the step at the other, governed by $EI d^4w/dx^4 = q(x)$ ¹⁵. For a continuous beam with a central point load the closed-form mid-span deflection is $\delta = PL^3/(48EI)$, which evaluates to 1.737 mm for the parameters above; this is the deflection of a perfectly rigid ramp, the best case that every telescopic layout is measured against. Two failure checks are applied. Strength is checked through the maximum bending stress $\sigma = M(h/2)/I$ against a target safety factor of 3, meaning the stress anywhere in the ramp must stay below one third of the material’s capability, a three-fold buffer for unexpected load such as extra weight or sudden impacts. Serviceability is checked against the standard deflection limit of span over 360²⁵, which for this span is $L/360 = 3.41$ mm. $L/360$ is a general structural criterion, historically set to protect rigid finishes such as plaster ceilings from cracking under floor beams. Neither IS 7480¹ nor the harmonised accessibility guidelines² specify a deflection limit for ramps, and no study located for this paper derives a deflection or slope-change threshold from wheelchair rollability directly, though rolling resistance on different surface types has been measured for wheelchairs elsewhere²⁶. $L/360$ is used here as the closest available general-purpose serviceability benchmark²⁷; Limitations returns to this gap.

The overlap joint model

Three kinds of joint bracket the possibilities. A rigid joint makes the two sections and the joint work together against the load as one piece. A pinned joint is like a door hinge: it holds the sections together but lets them rotate freely, so no bending moment crosses it. The overlap of a telescopic ramp is the third kind, a semi-rigid joint that carries some but not all of the moment⁸. The overlap acts as a rotational spring: when the inner section tilts, its left edge presses down into the outer wall and its right edge presses up into the opposite wall, and the outer section answers with a restoring torque that locks the mechanism in place. The two bearing contacts sit a lever arm L_{ov} apart, so a transmitted moment M is reacted by a force couple across the engagement length. With

a bearing stiffness k_c at each contact, the equivalent rotational stiffness of the joint is $k_r = \frac{1}{2}k_cL_{ov}^2$. The square law is the core physics: doubling the overlap multiplies the joint stiffness by four, because the lever arm and the displacement-to-rotation conversion each grow with L_{ov} . A long overlap therefore behaves as if welded, while a short one does not, because there is not enough lever arm to create the couple needed to tie the two sections together. The bearing stiffness was assumed as $k_c = Et = 1.38 \times 10^8$ N/m, the local stiffness scale of the $t = 2$ mm aluminium wall; this choice is stated openly as the one assumed constant in the model. This is a first-order stiffness scale, not a mechanical derivation of the bearing behaviour of two nested channel walls: $E \cdot t$ has the dimensions of a translational stiffness (N/m), but it is not obtained from a contact-mechanics analysis of the joint or from a test, in the way the semi-rigid connection literature cited above builds its moment-rotation models from experimental data⁵. k_c is also treated as a lumped bearing stiffness for the full 100 mm rail width, independent of that width — it is not a per-unit-width value that would need multiplying by b before entering $k_r = \frac{1}{2}k_cL_{ov}^2$. A wider or narrower rail would in reality change the contact area and hence k_c , an effect this single-constant model does not capture. “Sensitivity to the bearing-stiffness assumption” in the Discussion quantifies how far the design conclusion moves if k_c is wrong by a factor of two in either direction.

Numerical method

Because of the overlapping joints, the closed-form deflection formula no longer applies, so the beam is broken into small pieces called elements, each with two nodes. Each node can displace and rotate, two degrees of freedom, so each element carries four degrees of freedom and its stiffness matrix is the standard symmetric 4×4 Euler-Bernoulli element matrix^{15,28}, assembled by the direct stiffness method. Roughly 120 elements span the ramp (each about $L/120$ long, with extra nodes inserted at every joint and load position). A mesh-convergence check comparing 40, 80, 120 (the density used throughout this paper), 160, 240, and 480 elements across the same span found the reported deflection and stress unchanged to six decimal places at every density tested, so the mesh is well converged and not the limiting factor in any reported result. At each overlap joint the mesh uses a split rotation: the two sides share one vertical displacement but keep separate rotations, and the rotational spring k_r couples the two rotations, following the established treatment of beams with internal rotational discontinuities^{29,30}. The global stiffness matrix is assembled element by element, the simple supports impose zero displacement at both ends, and the system $Ku = F$ is solved directly. Mid-span deflection, the bending moment diagram, the maximum stress, and the safety factor are extracted from the solution.

Verification procedure

Verification means running the code for an answer already known from theory: if the code returns the same answer, the machinery contains no error. This is verification in the sense of Oberkampf and Trucano¹⁷: comparison against a closed-form mathematical solution, testing whether the code solves the assumed equations correctly. It is not validation, which would require comparison against a physical measurement; no such comparison was performed, and Future Work returns to this point. Three checks were required to pass with less than 1% error before the sweep. First, the rigid baseline with no joints must reproduce $\delta = PL^3/(48EI)$ and $M = PL/4$. Second, a beam with one overlap joint at mid-span given a near-rigid spring of $k_r = 10^{12}$ N·m/rad must reproduce the same continuous-beam answers, which tests the joint machinery itself, not just the plain beam. Third, because the first two checks bracket the joint stiffness at zero and near-infinity while every swept case and both critical-overlap crossings fall in the intermediate band of roughly 10^2 to 10^6 N·m/rad, a beam with one overlap joint at mid-span given a representative intermediate stiffness ($k_r = 2.5 \times 10^5$ N·m/rad, the value at the 60 mm design overlap) was checked against a closed-form solution obtained independently by the unit-load (Maxwell-Betti) method rather than against another run of the same code. Because the beam is statically determinate, the moment at the spring is always $PL/4$ regardless of k_r , so the relative rotation across the spring is $(PL/4)/k_r$, and by the unit-load theorem this adds an extra mid-span deflection of $PL^2/(16k_r)$ on top of the continuous-beam term: $\delta(k_r) = PL^3/(48EI) + PL^2/(16k_r)$. A deliberate-error test was also run: one support was removed on purpose to confirm the model detects an unstable structure rather than returning a plausible wrong number.

Parametric sweep and analysis

A parametric sweep keeps everything constant and varies one input to see how it alone affects the outcome. The overlap length was swept over 80 linearly spaced values from 2 mm to 150 mm, spanning from nearly disengaged to well past the designed 60 mm, across four layouts: a rigid baseline with no joints; a two-section ramp with one joint at $0.50L$; a three-section ramp with joints at $0.333L$ and $0.667L$; and a four-section ramp with joints at $0.25L$, $0.50L$, and $0.75L$, the layout of the real v9 design. This gives 320 cases. For each layout the critical overlap, the overlap at which mid-span deflection equals exactly 3.41 mm, was found by linear interpolation between adjacent sweep points. All 320 cases are recorded in a results table with the configuration, overlap, joint stiffness, mid-span deflection, maximum stress, and safety factor.

Ethical considerations

The study is entirely computational. No human or animal subjects were involved, and all input data come from the author's own design documentation and published material properties.

Results

Verification

The finite element model reproduced the closed-form solution exactly. The rigid baseline returned a mid-span deflection of 1.7370 mm against the closed-form 1.7370 mm and a maximum moment of 214.593 N·m against $PL/4 = 214.593$ N·m, an error of 0.0000% (agreement to double-precision floating-point machine epsilon, not literal mathematical exactness) on both quantities. The single-joint check with a near-rigid spring returned the same values, again with 0.0000% error, confirming that the split-rotation joint machinery reproduces the expected closed-form limit, not only the plain beam. A third check, at a representative intermediate stiffness of $k_r = 2.5 \times 10^5$ N·m/rad — the value at the 60 mm design overlap, well inside the 10^2 to 10^6 N·m/rad band every swept case and both critical-overlap crossings occupy — was compared against the independent closed-form solution derived in Methods ($\delta = PL^3/48EI + PL^2/16k_r = 2.00222$ mm); the FEM returned 2.00222 mm, again a 0.0000% difference. Five further spot checks spanning $k_r = 3 \times 10^2$ to 10^6 N·m/rad agreed to within 0.00002% in every case, closing the gap the first two checks left at zero and near-infinite stiffness. The agreement is exact to machine precision because Euler-Bernoulli elements are exact for point loads applied at nodes. These checks are verification in the sense of Oberkampf and Trucano¹⁷: they show the code solves the assumed equations correctly. They are not validation — none compares the model against a physical measurement of an actual overlap joint, and none is offered as one. In the deliberate-error test, removing one support made the stiffness matrix singular: without the right-hand support the structure is free to pivot downward about the remaining support under any applied load, which the solver reports as an unbounded deflection. A slight boundary-condition slip therefore cannot pass unnoticed.

Deflected shape

Figure 1 shows the four layouts and the position of each overlap joint. Figure 2 shows the deflected shape of each layout at a 20 mm overlap. The rigid baseline bends in a single smooth curve with its maximum of 1.74 mm at mid-span. The jointed layouts do not: at each overlap joint the slope changes sharply, and instead of a smooth curve the two-section ramp forms a sharp V shape right at the middle joint. This V shape is a kink, the visible signature of a joint too soft to carry the

bending moment across. At this overlap the two-section ramp deflects 3.96 mm, the three-section ramp 3.71 mm, and the four-section ramp 5.07 mm, all past the 3.41 mm limit.

Deflection versus overlap

Figure 3 shows mid-span deflection against overlap length for all four layouts. At the shortest overlap of 2 mm the joint stiffness is 276 N·m/rad and the deflections are 240.4 mm for the two-section, 213.5 mm for the three-section, and 359.8 mm for the four-section layout, between 120 and 210 times the rigid value of 1.737 mm. Deflection falls steeply as the overlap grows, following the square-law rise of joint stiffness; at the designed 60 mm overlap, where $k_r = 2.5 \times 10^5$ N·m/rad, the three layouts deflect 1.97 mm, 2.00 mm, and 2.13 mm respectively. All four curves ultimately intersect and meet when the overlap is long enough, around the 120 mm mark, and at 150 mm every layout lies within 4% of the rigid baseline. Deflections at the low-overlap end of this range are large enough — 20 to 30% of the 1.228 m span — that linear Euler-Bernoulli theory's small-deflection assumption (curvature approximated as w'' , reliable while the maximum beam slope stays under roughly 0.10 rad) no longer holds; that assumption fails below about 4 to 6 mm of overlap for the three jointed layouts (Table 1). The corresponding portion of the sweep, shaded in Figure 3, is numerically well-behaved but not a physically meaningful prediction: it also fails a first-order local bearing-stress check below about 4.5 mm of overlap (see “Local bearing stress at the overlap contacts” below), so no design conclusion in this paper relies on it. All three reported critical overlaps (22.5 to 29.3 mm) sit four to six times past the small-deflection threshold, deep inside the regime where the theory is reliable.

Critical overlap

The overlap at which each curve crosses the $L/360$ line of 3.41 mm is the critical overlap, shown per layout in Figure 5. The three-section ramp requires a minimum overlap of 22.5 mm, the two-section ramp 23.9 mm, and the four-section ramp 29.3 mm. The three-section layout, with two joints, needs a shorter overlap than the two-section layout, with one joint. The four-section layout needs the largest overlap of the three. All three critical overlaps lie below the 60 mm overlap specified in the v9 design, which is 2.0 times the largest of them.

Load position

Because the fixed mid-span load is justified above only by the continuous-beam argument, its validity for the jointed layouts was tested directly: for every layout, the load position was swept across the full span at overlaps of 5, 8, 12, 16, 20, and 60 mm, and the largest deflection anywhere along the beam (not only under the load) was recorded at each position. For

the two- and four-section layouts, which place a joint exactly at $0.50L$, mid-span remained the worst load position at every overlap tested, with zero gap to the next position tried. For the three-section layout, whose joints sit at $0.333L$ and $0.667L$, loading near a joint produced a marginally larger peak deflection elsewhere on the span at very short overlaps — 8.6% higher at 5 mm, 3.4% higher at 12 mm — but this gap closed to zero by 16 mm, comfortably below the 22.5 mm critical overlap reported for that layout. The mid-span assumption therefore holds across the whole range in which this paper's thresholds are read.

Stress and safety factor

Figure 4 shows the safety factor against overlap length. The maximum bending stress is 14.3 MPa in every one of the 320 cases, independent of layout and of overlap length, and the safety factor against the 240 MPa yield stress is flat at 16.8, far above the target of 3. Cutting the ramp into two, three, or four sections and overlapping them does not change the maximum internal bending moment, which stays at $PL/4 = 214.6$ N·m throughout; with the moment unchanged, the stress and the safety factor do not move.

Table 1 summarises the full 320-case sweep, reporting minimum, maximum, and mean deflection and stress per layout rather than only the highlighted overlaps discussed above.

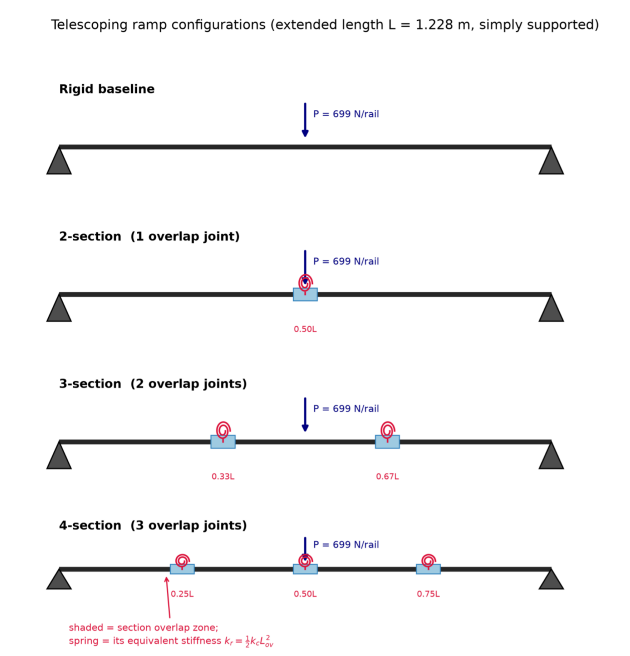


Figure. 1 The four ramp layouts: rigid baseline, two-, three-, and four-section telescopic configurations, with overlap zones and their rotational-spring representation.

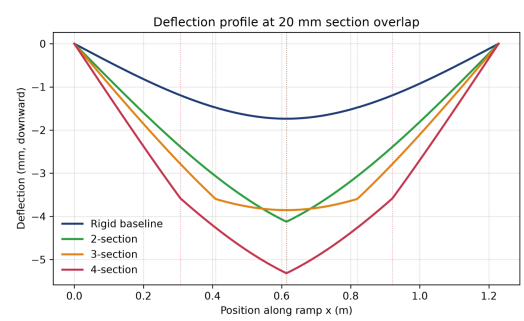


Figure. 2 Deflected shape of each layout at a 20 mm overlap. The jointed layouts kink at their overlap joints instead of bending in a smooth curve.

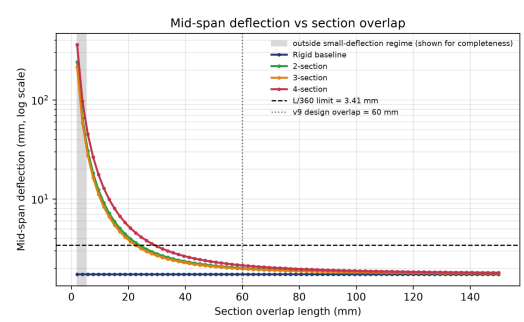


Figure. 3 Mid-span deflection against overlap length for all four layouts, with the L/360 serviceability limit (3.41 mm) and the 60 mm design overlap marked; the y-axis is logarithmic, and the shaded band below about 5 mm of overlap falls outside the small-deflection regime the model assumes and is shown for completeness only (see 'Deflection versus overlap').

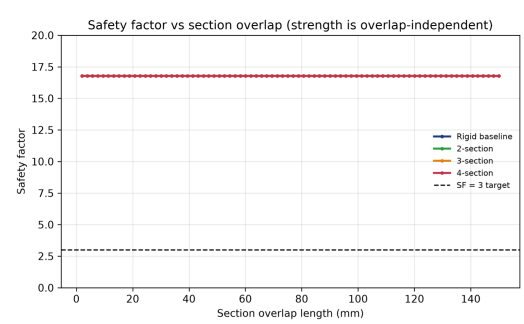


Figure. 4 Safety factor against overlap length. The safety factor is flat at 16.8 for every layout and every overlap, far above the target of 3.

Discussion

Key findings

The sweep answers both research questions with two results that were not obvious at the outset. First, the overlap con-

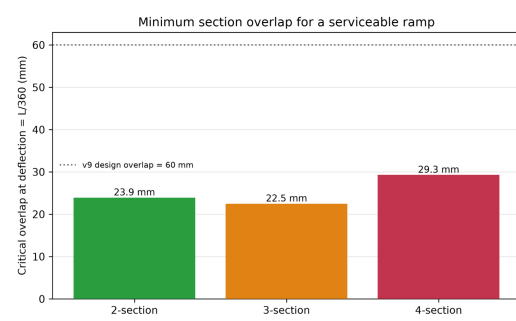


Figure. 5 Critical overlap per layout against the 60 mm design overlap of the v9 concept.

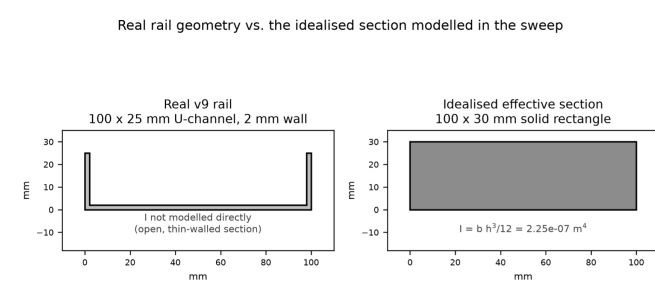


Figure. 6 The real v9 rail cross-section (100 x 25 mm U-channel, 2 mm wall) drawn to the same scale as the idealised 100 x 30 mm solid rectangle actually modelled in the sweep, added in response to review

Table 1 Summary statistics across the full 320-case sweep (80 overlaps $\times$ 4 layouts). Stress and safety factor are constant within each layout because the structure is statically determinate.

Configuration	$\delta_{\min}$ (mm)	$\delta_{\max}$ (mm)	δ_{mean} (mm)	$\sigma_{\max}$ (MPa)	SF
Rigid baseline	1.737	1.737	1.737	14.306	16.776
2-section	1.779	240.433	6.782	14.306	16.776
3-section	1.775	213.487	6.213	14.306	16.776
4-section	1.801	359.781	9.305	14.306	16.776

trols deflection and only deflection: across a 75-fold range of overlap the deflection changed by a factor of up to 210 while the stress did not move at all. Second, the hypothesis that more sections always means worse performance is wrong. The three-section ramp, with two joints, needs less overlap than the two-section ramp, with one.

Why overlap governs serviceability and not strength

The ramp rests on two supports and carries one load, so it is statically determinate: the bending moment diagram is fixed by equilibrium alone and the maximum moment is locked at $PL/4$ no matter how the joints behave. The joints add rota-

tional give, and that give appears as sag, not as extra fibre stress. The practical consequence is that a telescopic ramp of this kind does not fail by breaking. It fails by sagging past the point where it is compliant and feels safe, long before the aluminium is in any danger. The governing check for a telescopic ramp designer is therefore the deflection limit, and the safety factor, at 16.8 against a target of 3, establishes that the ramp holds the rated load without any risk of permanent bending.

Why joint position matters as much as joint count

Intuitively, it would seem that the more the sections, the larger the needed overlap must be. That is not what the data show, and the reason is the placement of the joint relative to the load. The two-section layout puts its single joint at $0.50L$, directly under the mid-span load, exactly where the bending moment and the rotation demand are highest. The three-section layout keeps both of its joints away from mid-span, at $0.333L$ and $0.667L$, where the moment is lower, so each joint is asked to do less and a shorter overlap suffices. The four-section layout is the worst of the three because the two effects stack: it has a joint under the load and it has the most joints. The simple idea that more sections is always worse is not right; what degrades the ramp is joints at the point of peak moment, compounded by their number. Every layout compared so far, however, is an evenly spaced standard grid, so joint count and joint position change together; “Isolating the position effect from the count effect” below breaks that link directly.

Connection to the design

The four-section v9 concept specifies a 60 mm overlap, and its critical overlap from the sweep is 29.3 mm, so the design carries a two-fold serviceability margin and sits well inside the safe zone for the idealised 100 x 30 mm surrogate section modelled here. Because that section was fixed, before the sweep, to make the rigid baseline serviceable rather than measured from the real 100 x 25 mm channel (see “Geometry, material, and load”), this margin should be read accordingly. This is an independent confirmation, from first-principles mechanics, applied to a representative surrogate geometry, of a dimension that was originally chosen by judgement, and not a certified safety margin for the physical channel section itself, whose true stiffness was not computed. Two recommendations follow for telescopic ramp design in general. The minimum overlap should appear on the manufacturing drawing as a hard dimension, with end-stops set so the sections cannot extend past it in service, because a user who over-extends the ramp is silently removing its stiffness. And where the layout allows, an overlap joint should not be placed directly under the point of heaviest load; a joint kept off-centre costs less serviceability than one under the load.

Isolating the position effect from the count effect

The four-section, three-section, and two-section layouts compared above are all evenly spaced standard grids, so every layout that has more joints also happens to have a joint closer to mid-span, and joint count and joint position are confounded in that comparison. To separate them, a fourth 4-section layout was swept with the same three joints and the same $0.25L$ spacing between them, but the whole grid shifted so that no joint sits at $0.50L$ (joints at $0.35L$, $0.60L$, $0.85L$ instead of $0.25L$, $0.50L$, $0.75L$). Its critical overlap is 26.4 mm, against 29.3 mm for the standard layout with the same joint count and spacing but a joint under the load — a roughly 10% reduction from moving the joints off-centre alone, with the number of joints held fixed. This isolates the position effect the previous section infers from a between-layout comparison and confirms it directly: it is where the joints sit, not only how many there are, that sets the critical overlap.

Sensitivity to the bearing-stiffness assumption

Because k_c is assumed rather than measured (see “The overlap joint model”), and every critical-overlap number in this paper scales with it, the full sweep was rerun with k_c halved and doubled. Table 2 reports the result. Halving k_c raises the three critical overlaps by about 40 to 45%, to 33.8, 31.8, and 41.4 mm; doubling it lowers them by about 30%, to 16.9, 16.0, and 20.7 mm. Even in the worst case tested across this range — k_c halved, four-section layout, critical overlap 41.4 mm — the 60 mm v9 design overlap still clears it, with a 1.45x margin. The design conclusion is therefore not fragile to a factor-of-two error in the one assumed constant, though the exact margin quoted elsewhere in this paper (2.05x for the four-section layout) is specific to the assumed $k_c = Et$ and should be read as a point estimate within this wider range.

Table 2 Critical overlap and 60 mm design margin under the assumed bearing stiffness $k_c = Et$, and under k_c halved and doubled.

Configuration	k_c halved: crit. overlap (mm) / margin	Baseline k_c : crit. overlap (mm) / margin	k_c doubled: crit. overlap (mm) / margin
2-section	33.8 / 1.78×	23.9 / 2.51×	16.9 / 3.55×
3-section	31.8 / 1.89×	22.5 / 2.67×	16.0 / 3.75×
4-section	41.4 / 1.45×	29.3 / 2.05×	20.7 / 2.90×

Local bearing stress at the overlap contacts

The joint model already implies a bearing force at each contact: since the transmitted moment M is reacted by a force couple with lever arm L_{ov} (see “The overlap joint model”), the contact force is $F = M/L_{ov}$, and $M = PL/4 = 214.6 \text{ N}\cdot\text{m}$ is constant everywhere in the sweep. Dividing F by an assumed

contact patch equal to the full rail width and wall thickness ($100\text{ mm} \times 2\text{ mm} = 200\text{ mm}^2$) gives a first-order bearing stress. At the reported critical overlaps this stress is 37 to 48 MPa, 15 to 20% of the 240 MPa yield stress; at the 60 mm design overlap it is 18 MPa, 7.5% of yield. Both are comfortably below yield, supporting the claim elsewhere in this paper that strength does not govern in the serviceable range. Below about 4.5 mm of overlap, however, this estimate exceeds the 240 MPa yield stress outright (536 MPa at the 2 mm sweep minimum, more than double yield), meaning that portion of the sweep is invalidated by local bearing failure as well as by the large-deflection breakdown noted above — a second, independent reason that low-overlap region is shown for completeness only and carries no design conclusion.

Point-spring versus distributed overlap idealisation

The overlap is modelled as a single zero-length rotational spring at one point, even though the physical engagement spans a finite length L_{ov} . To bound how much this idealisation matters, each point spring in the four-section layout — the layout with the sharpest kink in Figure 2 — was replaced with a proxy pair of springs at the two physical ends of the engagement length, each twice as stiff, with the ordinary beam element bridging the overlapped stub of rail between them. This is not a physical model of bearing contact, only a bound on the simplification. At overlaps of 20, 29.3, and 60 mm, the distributed proxy deflects 1.1 to 1.5% less than the point-spring model — the point-spring idealisation is, if anything, mildly conservative rather than optimistic, and the effect is small next to the sensitivity to k_c documented above.

Limitations

The model is linear-elastic Euler-Bernoulli beam theory with a static point load held at mid-span, the worst case for deflection but not a full description of a wheelchair rolling along the span, though the load-position sweep in Results confirms mid-span governs across the range this paper's thresholds are drawn from. The overlap is condensed into a single rotational spring through the force-couple model, with the bearing stiffness assumed as $k_c = Et$ rather than measured; the critical overlaps scale with that constant, so they should be read as model estimates, not certified values. A halving or doubling of k_c moves the three critical overlaps by roughly +40% to -30% (Table 2), and even in the worst case tested the 60 mm design overlap still clears the critical overlap, with a 1.45x margin. Contact friction, local crushing of the thin wall, and the perforated base of the real section are all outside the model, though a first-order bearing-stress check suggests local crushing is not governing in the serviceable range (see “Local bearing stress at the overlap contacts”). The effective

30 mm section depth was fixed so the rigid baseline is serviceable, which makes the comparison between layouts clean but means the absolute deflections of the real channel section will differ, and the roughly two-fold design margin reported above belongs to this idealised surrogate section, not to a certified analysis of the physical channel. The L/360 serviceability threshold itself is a general structural criterion, not one derived for wheelchair rollability specifically; a ramp- and wheelchair-specific deflection or slope-change criterion is future human-factors work this study does not attempt. Nor has the joint law $k_r = \frac{1}{2}k_c L_{ov}^2$ been checked against a physical measurement of an actual overlap joint — every verification check in this paper compares the code against closed-form mathematics, not against hardware (see “Verification procedure”). Because of that, every absolute overlap threshold reported in this paper — the three critical overlaps, the bearing-stress estimates, and the two-fold design margin — should be read as the output of an assumed mechanical model, not as a validated design value, until the bench test proposed below is carried out. These simplifications are the price of isolating one variable cleanly, and they mark the boundary of what the numbers claim.

Future work

The 900 N case has since been run directly: raising the rated load from 699 N to 900 N raises the three critical overlaps to 32.4 mm (two-section), 30.5 mm (three-section), and 39.6 mm (four-section) — each larger than its 699 N counterpart, as expected, and every one of them still below the 60 mm v9 design overlap, with a margin of 1.5x in the worst case. The safety factor at 900 N is 13.0, still far above the target of 3. The stronger extension is physical: a bench test of two overlapping channel sections under a central load would measure the real moment-rotation behaviour of the overlap, recovering k_c directly at overlaps of 20, 30, 40, and 60 mm from $k_r = M/\Delta\theta$ given an applied moment and the resulting relative rotation, testing both the assumed L_{ov}^2 scaling and the magnitude of k_c , and would turn the model estimates into certified minimums.

Closing Thought

A telescopic ramp lives or dies by a dimension that never appears in accessibility codes: the length over which its sections engage. This study shows that the dimension can be modelled with nothing more than beam theory and a rotational spring, that it controls serviceability rather than strength, and that where the joints sit matters as much as how many there are. The result is a floor a designer can enforce, an overlap length written on a drawing, so that a ramp that folds flat for the boot of a car still feels like solid ground under a wheelchair.

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