

Estimating Earth's Temperature Response with Transformed and Augmented OLS

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The long-term relationship between radiative forcing and surface temperature is central to predicting the impacts of climate change. This study employs multicointegration to characterize this relationship and provides the first application of the Transformed and Augmented Ordinary Least Squares (TAOLS) estimator to estimate the multicointegration model. The main objective is to estimate the Equilibrium Climate Sensitivity (ECS), defined as the global mean surface temperature increase following a doubling of atmospheric carbon dioxide. Diagnostic tests reveal that radiative forcing innovations are strongly non-Gaussian, providing a key motivation for applying semiparametric TAOLS rather than the parametric Gaussian maximum likelihood method. TAOLS is also robust to misspecification of the short-run dynamics. Using the three data pairings of bru^1 , we obtain TAOLS estimates of ECS ranging from 1.81°C to 2.49°C , consistently below their main maximum likelihood estimate of 2.80°C . Because the model we use is linear and is based on global-mean data, it cannot capture state-dependent feedbacks or regional differences; extending TAOLS along either dimension is a natural next step.

Keywords: Climate Sensitivity, Cointegration, Multicointegration, Radiative Forcing, Surface Temperature,

Introduction

This paper studies the long-term relationship between radiative forcing and Earth's surface temperature. Radiative forcing represents the net change in the energy balance of the Earth system due to external factors and is measured as the difference between incoming solar radiation and outgoing thermal energy. This energy imbalance arises primarily from changes in the concentrations of greenhouse gases such as methane (CH_4) and carbon dioxide (CO_2). Surface temperature refers to the air temperature measured at approximately two meters above Earth's surface. While surface temperature responds to changes in radiative forcing, the response is not immediate. Such a delay is attributed to the oceans' very slow heating rate, as it takes four times as much thermal energy for a unit mass of water to heat up as air. With water covering approximately 71% of Earth's surface, oceans absorb between 89% and 91% of the excess energy added to the Earth system due to positive radiative forcing from greenhouse gases, as estimated by von Schuckmann et al.² and the Intergovernmental Panel on Climate Change³ p. 925. This leads to a delayed thermal response that may last from decades to centuries, which complicates efforts to predict long-term climate outcomes.

This paper focuses on a particular long-term climate prediction: how much the global surface temperature would eventually rise if atmospheric CO_2 were doubled from pre-industrial

levels (circa 1850) and the climate system reached equilibrium. This increase, which accounts for all major climate feedbacks such as water vapor, ice-albedo, and clouds, is defined as the Equilibrium Climate Sensitivity (ECS), a central concept in climate science with practical effects on climate policy. Specifically, it refers to the warming from a doubling of pre-industrial CO_2 concentrations (~ 280 parts per million (ppm) to 560 ppm) once the system reaches equilibrium. For context, the atmospheric CO_2 concentration was approximately 424 ppm in 2024⁴.

To estimate the ECS, we need to characterize the long-run relationship between Earth's surface temperature and radiative forcing. These two time series share the same fundamental statistical characteristics as economic time series. Both exhibit persistent trending behavior, whether deterministic or stochastic, and both are driven by the cumulative effects of structural forces over time. In addition, each represents only a single historical realization over a period that is short relative to the time scales of interest. It is therefore natural to model both radiative forcing and surface temperature as integrated processes, as is typically done in econometrics. A process is called integrated if it can be represented as the cumulative sum of largely independent shocks; here, cumulative summation is the discrete analog of integration. Cointegration and multicointegration are the canonical models for this situation: they capture a stable long-run relation between two series although both series may be unpredictable on their own.

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The concept of cointegration was introduced and developed by Nobel laureates Robert Engle and Clive Granger⁵. While each time series can drift or fluctuate independently, cointegration constrains them to move together in the long run. Cointegration goes hand-in-hand with the error-correction mechanism (ECM), which connects short-term fluctuations to prior deviations from the long-term balance. As an example, imagine there are two boats floating in a lake, connected by an elastic rope. Although currents may pull the boats in different directions, the rope's tension will pull the boats back to equilibrium. Thus, their future movements are shaped by their current separation.

Multicointegration, an extension of cointegration developed by Granger and Lee^{6,7}, builds upon the original concept by examining how cumulative sums of past imbalances can shape the system's behavior over longer periods. Going back to the boat example, multicointegration additionally accounts for how past "stretching" of the system can affect future movements, capturing long-term feedbacks.

From the perspective of climate science, cointegration excels at modeling interactions between radiative forcing and temperature, while multicointegration is crucial for modeling the delayed effects of accumulated oceanic heat content on surface warming. While radiative forcing and temperature may each follow integrated trajectories with local variations, cointegration can uncover a long-run connection between the two, indicating they co-move over time. Multicointegration adds to this analysis by exploring how the cumulative energy stored in the oceans influences surface temperature. Since the ocean is a slow-responding reservoir of thermal energy, it absorbs heat over decades. The deep ocean then gradually releases this stored energy to the atmosphere over centuries, slowly raising surface temperature. These physical dynamics have a direct formal counterpart in the econometric framework: *pre*⁸ shows that a two-component energy balance model (EBM) is mathematically equivalent to a cointegrated vector autoregression where the climate feedback parameter is identifiable as a cointegrating coefficient, so that cointegration is not merely a statistical metaphor imported from economics but the direct econometric representation of the physical energy balance of the climate system.

Econometric modeling of climate data offers some advantages over physical simulation models such as general circulation models (GCMs). As argued by *hil*⁹, econometric methods "do not depend on the accuracy of any complex global climate model" and instead extract the long-run signal from the observed statistical properties of the data. In contrast, GCMs calibrate key parameters such as ECS using model simulations. Econometric methods estimate these parameters directly from data, providing independent, data-driven estimates that complement GCM-based projections without imposing all of their structural assumptions.

Among existing econometric approaches, the Maximum Likelihood Estimation (MLE) of Johansen¹⁰ within an ECM framework has been widely used to estimate the long-run relationship among climate time series. For instance, Bruns, Csereklyei, and Stern¹ (hereafter BCS (2020)) apply MLE within a multicointegrating model. Although MLE is asymptotically efficient under correct specification, it has two key limitations in our context. First, it imposes a Gaussian distribution on the model innovations. This assumption is likely to be violated when the data contain large non-Gaussian spikes, such as those induced by major volcanic eruptions. Second, it requires correct specification of the short-run VAR dynamics, which is often practically infeasible. This study applies the Transformed and Augmented Ordinary Least Squares (TAOLS) method of Sun, Phillips, and Kheifets¹¹ (hereafter SPK (2025)). TAOLS is more robust than MLE because it does not impose Gaussianity on the model innovations and does not require precise specification of the short-run VAR dynamics. By applying TAOLS to the same dataset from BCS (2020), this paper provides a direct comparison of TAOLS and MLE estimates of ECS.

We find that the TAOLS estimates of ECS are consistently lower than the corresponding MLE estimates reported in BCS (2020), with each MLE ECS estimate exceeding the upper bound of the corresponding 95% TAOLS confidence interval. Quantitatively, while the MLE yields a point ECS estimate of 2.80°C for a focal case, the TAOLS method produces a range of 1.81°C to 2.49°C. The precise value of ECS remains a subject of extensive debate within the climate research community (cf. Roe and Baker¹²). Some studies propose an ECS below 2°C (e.g., Lewis and Curry¹³), whereas others indicate values exceeding 5°C (e.g., Bjordal et al.¹⁴). In this context, by applying a more robust estimator to the same dataset as BCS (2020), this study contributes to this discourse by providing empirical evidence for a moderate ECS.

Related Literature. Several broad methodological approaches have been developed to estimate ECS. EBMs, originating with Budyko¹⁵ and Sellers¹⁶, relate global-mean temperature change to radiative forcing through a scalar feedback parameter; *gre*¹⁷ show how this parameter can be inferred from GCM regression experiments. *lew*¹³ apply the EBM framework to the AR5 observational record and obtain an ECS of approximately 1.64°C, a low estimate attributable to the "pattern effect"³: the effective feedback parameter changes as ocean heat uptake patterns evolve, so short-record EBM estimates tend to understate the true equilibrium response. By contrast, *bjo*¹⁴ show, using cloud-resolving model simulations, that state-dependent cloud feedbacks could push ECS above 5°C. The IPCC Sixth Assessment Report³ synthesizes process-based evidence, the instrumental record, and paleoclimate reconstructions to argue for a likely range of 2.5–4.0°C. In the econometrics literature, *kau*¹⁸ apply cointegra-

tion techniques to hemispheric temperature relations; kau¹⁹ provide econometric support for the $I(1)$ characterization of global temperature. As noted above, pre⁸ validates the multicointegration framework of bru¹ by establishing the formal equivalence between EBM and cointegrated vector autoregressions. phi²⁰ employ dynamic panel cointegration to estimate the transient climate sensitivity from station-level data. The multicointegration approach of BCS (2020), to which we apply TAOLS, directly models the accumulation of ocean heat content and estimates the long-run equilibrium response, thereby avoiding the transient-vs.-equilibrium conflation inherent in shorter-run EBM methods.

The remainder of this paper is organized as follows. The next section lays out the theoretical framework, followed by the TAOLS methodology. The empirical study is then presented, and the final section provides concluding remarks and discusses future research directions.

Theoretical Framework: Cointegration and Multicointegration

Cointegration

We now formalize the statistical framework sketched in the introduction, beginning with the definition of integrated processes. In the simplest setting, a time series $\{x_t\}$ is integrated if it can be represented as the sum of an independently and identically distributed (i.i.d.) sequence $\{u_t : Eu_t = 0\}$

$$x_t = \sum_{s=1}^t u_s$$

for all $t = 1, \dots, T$, where T is the length of the time series. In this formulation, x_t represents a “random walk,” where each new value is the accumulation of all previous shocks $\{u_t\}$. It is clear that the variance of x_t grows with t , and the process is hence non-stationary. Intuitively, the process does not revert to a fixed mean; instead, it tends to wander away from its current position, as the impact of every historical shock is permanent.

More generally, the zero-mean shocks $\{u_t\}$ can be correlated over time, in which case we obtain a general integrated process. Since there is only one summation involved in the definition of x_t , such a process is called integrated of order one, and we denote this as $x_t \sim I(1)$. An $I(1)$ process is also referred to as a unit root process, as its autoregressive representation contains a root equal to one. On the other hand, we write $u_t \sim I(0)$ to signify that it is not integrated. An $I(0)$ process has a fixed mean and variance and is called stationary; it tends to revert to its mean over time.

If two time series are individually $I(1)$ processes, but their linear combination is an $I(0)$ process, then they are cointegrated. As an example, consider the time series of personal income (x_t) and consumption expenditure (y_t). Both series

are affected by stochastic shocks, making them non-stationary. However, they remain tied together by a long-run budget constraint. If there exists a coefficient β such that $y_t - \beta x_t$ is stationary, then the two series are cointegrated. This directly implies a long-run equilibrium relationship despite short-term fluctuations:

$$y_t = \beta x_t + e_t$$

where e_t is a stationary $I(0)$ error term with a constant mean and finite variance.

For example, in climate science, radiative forcing (f_t) and surface temperature (s_t) exhibit similar statistical characteristics. Increases in greenhouse gases and other forcing agents alter the Earth’s energy balance, causing radiative forcing to behave as a non-stationary $I(1)$ process. Surface temperature adjusts to these changes over time and likewise exhibits non-stationary $I(1)$ behavior. If a coefficient λ exists such that

$$q_t = f_t - \lambda s_t$$

is stationary, then f_t and s_t are cointegrated. Here, λ plays the role of the cointegrating coefficient β in the general case, and q_t represents the equilibrium error, interpreted physically as the instantaneous change in Earth’s heat content (i.e., the net energy absorbed or released by the climate system). This cointegrating relationship suggests that radiative forcing imposes a long-term constraint on surface temperature.

An appealing characteristic of cointegration is its robustness to short-lived noise and temporary deviations from the long-run relationship. For example, aerosol forcing may be unusually high in a particular year, temporarily depressing the surface temperature signal, s_t . Even so, the underlying relationship with radiative forcing, f_t , may remain intact. As the temporary disturbance fades, surface temperature converges back toward its long-run equilibrium with radiative forcing.

Multicointegration

Multicointegration goes beyond cointegration by incorporating cumulative sums of the equilibrium errors into a second layer of long-term dependence. Equilibrium errors measure the short-term deviations of the variables from their long-term cointegration relationships. As an example, consider the previous analogy with income and expenditure. Let us assume that income and spending are cointegrated. In a real-world scenario, debt and savings also exist and they represent the cumulative sum of past deviations between income and spending. For example, given that a household consistently spends more than its income, it will naturally accumulate debt. Conversely, if it consistently spends less, it will accumulate savings. These accumulated stocks will affect future decisions regarding income and spending. Since the system is governed by both the contemporaneous flow relationship and the cumulative effects of past imbalances, it exhibits multicointegration.

In the climate domain, multicointegration manifests when the cumulative heat content ($Q_t = \sum_{\tau=1}^t q_\tau$), derived from the cointegrating relationship $f_t = \lambda s_t + q_t$, establishes its own equilibrium with surface temperature (s_t). Here, Q_t represents the total cumulative energy imbalance of the climate system—encompassing the oceans (the dominant reservoir), land, ice sheets, and the atmosphere. Following BCS (2020), we use ocean heat content as the empirical proxy for Q_t , since the oceans absorb the overwhelming majority of the accumulated imbalance; we acknowledge this is an approximation, as other reservoirs also contribute to the total energy stored. This stored energy influences surface temperature through processes such as ocean heat uptake and release, spanning time periods far longer than regular weather cycles. Therefore, there may also be a long-run relationship between Q_t and s_t . Define

$$v_t = Q_t - \phi s_t.$$

Since Q_t and s_t are both $I(1)$ processes, if there exists a constant ϕ such that v_t is stationary, then by definition, Q_t and s_t are cointegrated. In this case, a multicointegrated system is formed with two distinct equilibrium relationships: one between f_t and s_t , and another between their cumulative equilibrium error (Q_t) and s_t .

The importance of this second layer of cointegration is particularly evident in the climate context. For example, a permanent rise in radiative forcing due to industrial emissions could increase f_t over decades and drive s_t upward while also building up Q_t in the deep ocean. Over centuries, this stored oceanic heat is gradually transferred to the atmosphere, amplifying s_t and creating a long-run feedback that multicointegration captures.

BCS (2020) apply the multicointegration framework within a parametric ECM using the MLE method of Johansen¹⁰. Such an approach is effective at capturing multiple long-run equilibria. However, it requires explicit parameterization of short-run dynamics. For example, it has to specify how the change in surface temperature Δs_t this year is related to q_{t-2} , the heat content flux two years earlier. Furthermore, it imposes Gaussianity on the VAR innovations, which is not plausible in our study, as we show in the empirical section.

Methodology: TAOLS Approach

The TAOLS method is a semiparametric estimator designed to estimate multicointegrating relationships with greater flexibility and reduced computational burden compared to the MLE method of Johansen¹⁰. We begin with the foundational cointegrating relationship:

$$f_t = \lambda s_t + q_t, \quad (1)$$

where f_t is radiative forcing (in watts per square meter), s_t is surface temperature (in degrees Celsius), λ is the cointegrating coefficient, and q_t is the stationary error representing heat content flux. Cumulative sums are then defined as

$$F_t = \sum_{\tau=1}^t f_\tau, \quad S_t = \sum_{\tau=1}^t s_\tau, \quad \text{and} \quad Q_t = \sum_{\tau=1}^t q_\tau.$$

Taking cumulative sums of equation (1) and substituting $Q_t = \phi S_t + v_t$, we obtain the multicointegrating regression:

$$F_t = \lambda S_t + Q_t = \lambda S_t + \phi S_t + v_t.$$

Here, λS_t reflects the cointegration between cumulative forcing and cumulative temperature, ϕS_t represents the multicointegrating feedback from stored heat to current temperature, and v_t represents unobserved random fluctuations.

Incorporating a constant and a linear time trend into the basic multicointegration model, we specify the empirical model as

$$F_t = \gamma + \mu t + S_t \lambda + s_t \phi + v_t, \quad (2)$$

where the constant γ and linear trend μt arise naturally from the multicointegration structure: including an intercept in the level equation (1), or equivalently in the lower-tier relation $Q_t = \phi S_t + v_t$, generates a linear trend in the cumulative equation upon summation. The linear trend therefore allows for the possibility of a deterministic trend without imposing one—if no such trend is present in the data, the estimated coefficient $\hat{\mu}$ will be indistinguishable from zero. Just as an intercept is routinely included in a regression to avoid misspecification, including the linear trend here is the appropriate default.

Following the framework of SPK (2025), we implement the TAOLS method through a three-stage process. First, we augment the model with the first difference Δs_t in order to address endogeneity: long-run correlations between v_t and Δs_t would otherwise bias the estimates of λ and ϕ , since short-term fluctuations—such as a 0.3°C El Niño spike—would contaminate the long-run estimates. The augmented model is

$$F_t = \gamma + \mu t + S_t \lambda + s_t \phi + \Delta s_t \delta + e_t. \quad (3)$$

Second, we project all variables onto a set of low-frequency basis functions. Specifically, we use the following sine functions:

$$\left\{ \phi_i(r) = \sqrt{2} \sin \left(\left(i - \frac{1}{2} \right) \pi r \right) : i = 1, \dots, K \right\}.$$

Here $r = t/T \in [0, 1]$ is a normalized time index. In principle, any complete orthogonal basis can be used; the key requirement is that the first K basis functions have their energy concentrated at low frequencies, so that the transformation captures the long-run variation of the underlying integrated

series. Sine functions, cosine functions, and Fourier (sine-cosine) pairs all satisfy this requirement. Following PK2²¹, we adopt the above orthonormal sine basis.

The corresponding transformed variables are:

$$\begin{aligned} V_{F,i} &= \frac{1}{\sqrt{T}} \sum_{t=1}^T F_t \varphi_i\left(\frac{t}{T}\right), V_{S,i} = \frac{1}{\sqrt{T}} \sum_{t=1}^T S_t \varphi_i\left(\frac{t}{T}\right), \\ V_{s,i} &= \frac{1}{\sqrt{T}} \sum_{t=1}^T s_t \varphi_i\left(\frac{t}{T}\right), V_{\Delta s,i} = \frac{1}{\sqrt{T}} \sum_{t=1}^T \Delta s_t \varphi_i\left(\frac{t}{T}\right), \\ V_{\ell,i} &= \frac{1}{\sqrt{T}} \sum_{t=1}^T (1,t) \varphi_i\left(\frac{t}{T}\right), V_{e,i} = \frac{1}{\sqrt{T}} \sum_{t=1}^T e_t \varphi_i\left(\frac{t}{T}\right). \end{aligned} \quad (4)$$

These transformed variables retain the low-frequency components of the time series while filtering out high-frequency variation. The parameter K denotes the number of basis functions used in the transformation. Based on the transformed variables, we now have

$$V_{F,i} = V_{\ell,i} \alpha + V_{S,i} \lambda + V_{s,i} \phi + V_{\Delta s,i} \delta + V_{e,i} \quad (5)$$

for $\alpha = (\gamma, \mu)'$ and $i = 1, 2, \dots, K$. This is our transformed and augmented regression model.

Third, we apply OLS to the transformed regression model (5) to obtain estimates of $\gamma, \mu, \lambda, \phi$, and δ . Unlike Johansen's MLE, which requires a fully specified ECM and assumes normality, TAOLS relies on the asymptotic properties of the transformed series, delivering estimators that are consistent and asymptotically mixed-normal under general conditions (e.g., weak dependence in e_t). A further advantage of TAOLS is that standard inference methods, such as the t -test, can be readily applied, making statistical inference straightforward. For details on the asymptotic theory, see SPK¹¹.

Results

Data Sources

The empirical study uses data from bru¹, which were obtained from multiple sources and span the period 1850–2014, yielding $T = 165$ annual observations, as detailed in Appendix A of bru¹. Radiative forcing is computed using an established formula for seven components: well-mixed greenhouse gases, solar irradiance, tropospheric sulfate aerosol, black carbon, organic carbon, ozone, and stratospheric aerosol. For example, the radiative forcing of carbon dioxide (CO₂), a critical greenhouse gas in climate research, is calculated using the logarithmic relationship:

$$RF_{CO_2} = 5.35 \times \ln\left(\frac{C}{C_0}\right), \quad (6)$$

where C is the CO₂ concentration in ppm, and C_0 is the pre-industrial level (1850), approximately 280 ppm. In the

above, 5.35 is an empirically derived coefficient from myh²² that converts a change in CO₂ concentration into a change in Earth's radiative energy balance. We consider two versions of aggregate radiative forcing: full-efficacy radiative forcing (hereafter TotalRF) and partial-efficacy radiative forcing (hereafter MarvelRF), the latter adjusting the forcing from ozone, volcanic aerosols, and solar irradiance by a factor of 0.5.

Surface temperature data are sourced from Berkeley Earth²³ and the Hadley Centre/Climatic Research Unit (HadCRUT)²⁴. Temperatures, reported in degrees Celsius, are expressed as anomalies relative to the January 1951–December 1980 average.

Figure 1 reproduces Figures 1 and 2 of bru¹, to provide a visual overview of the data. Panel (a) plots both radiative forcing series and panel (b) plots both surface temperature series. Radiative forcing and surface temperature display evident comovement. However, the presence of large spikes induced by volcanic eruptions suggests that radiative forcing is unlikely to conform to a normal distribution. Consequently, applying a VAR system under the normality assumption, as implemented in bru¹, may yield unreliable estimates. The differences between the two versions of each series indicate some uncertainty in their measurement. If these differences are of a high-frequency nature, then TAOLS is particularly well-suited, as it does not rely on the high-frequency components of the underlying time series.

Stochastic Trend and Multicointegration Assumptions

The TAOLS framework requires that radiative forcing f_t and surface temperature s_t each be integrated of order one ($I(1)$) and that the system be multicointegrated. These properties are supported by formal unit-root tests, physical reasoning, and the system-level rank tests reported by bru¹.

Unit-root $I(1)$ tests. We apply two complementary unit-root tests to all four series—TotalRF, MarvelRF, Berkeley Earth temperature, and HadCRUT temperature—over the full sample 1850–2014 ($T = 165$). The Augmented Dickey–Fuller (ADF) test²⁵ has the null hypothesis of a unit root; failure to reject is evidence of $I(1)$ behavior. The KPSS test²⁶ has the null hypothesis of trend stationarity; rejection is evidence of a unit root. A pattern of ADF non-rejection combined with KPSS rejection thus constitutes strong evidence in favor of $I(1)$. For the ADF test, we include a constant (no deterministic trend) and select the lag length by BIC, up to a maximum of three lags. For the KPSS test, we use an automatic bandwidth selection procedure and test the null of trend stationarity. Table 1 reports the results.

For all four series, the ADF statistic is well above the 5% critical value of -2.87 , so the null of a unit root is not rejected at any conventional significance level. Symmetrically, all four

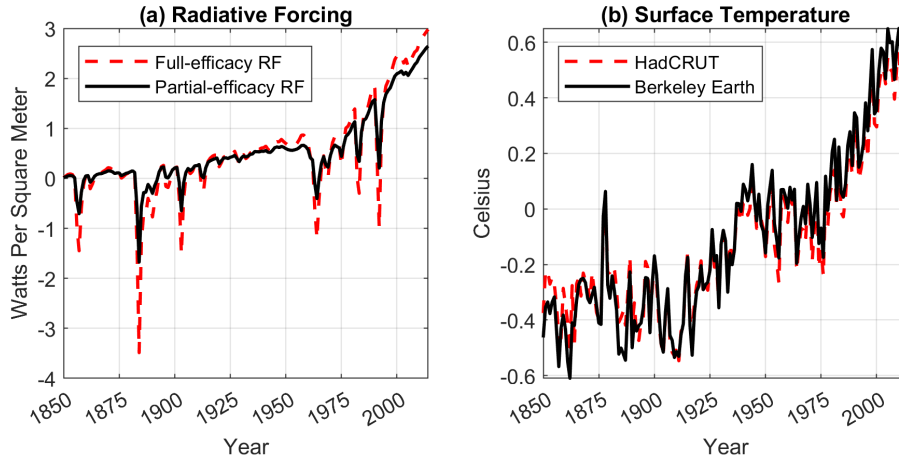


Figure. 1 Radiative forcing (panel a) and surface temperature anomaly (panel b), 1850–2014. Panel (a): full-efficacy and partial-efficacy radiative forcing (W m^{-2}). Panel (b): Berkeley Earth and HadCRUT surface temperature anomalies ($^{\circ}\text{C}$, relative to 1951–1980).

Table 1 Unit-Root Test Results (1850–2014, $T = 165$)

Series	ADF: Constant		KPSS: Trend	
	Stat (lag)	p -val	Stat	p -val
TotalRF	−1.653 (2)	0.449	0.921	≤ 0.010
MarvelRF	−0.179 (2)	0.938	1.667	≤ 0.010
Berkeley	−0.073 (3)	0.950	1.481	≤ 0.010
HadCRUT	−0.098 (3)	0.947	1.635	≤ 0.010

Notes: ADF H_0 : unit root; lag by BIC (max 3); 5% CV = −2.87. KPSS H_0 : trend stationary; 5% CV = 0.146, 1% CV = 0.216.

KPSS statistics far exceed the 1% critical value of 0.216, so the null of trend stationarity is strongly rejected. The two tests agree: every series is $I(1)$.

The $I(1)$ characterization can also be based on physical reasoning and empirical evidence from the broader econometric literature. Radiative forcing is driven by the slow accumulation of greenhouse gas concentrations from persistent anthropogenic sources; surface temperature adjusts to forcing changes with a multi-decadal lag governed by ocean thermal inertia. As argued by bru¹ following kau²⁷, absent radiative forcing the climate system would be mean-reverting; it is because forcing itself follows a stochastic trend that temperature does likewise. kau¹⁹ directly evaluate conflicting statistical claims about the integration order of surface temperature, including arguments in favor of trend-stationary representations, and conclude that the data cannot reject the unit-root characterization once the analysis is conducted carefully.

Moreover, bru¹ note, following kej²⁸, that a unit-root pro-

cess is the limiting case of a trend-stationary process with a structural break every period: with enough breakpoints any random walk can be approximated by a piecewise linear trend-stationary process, making the two representations nearly observationally equivalent in samples of this length. bru¹ conduct Johansen $I(2)$ rank tests on the identical dataset and sample period, concluding that the two series are cointegrated and the system is multicointegrated. Since our analysis uses the same data and sample, these results carry over directly.

Normality of innovations. The Jarque–Bera (JB) test²⁹ examines whether the skewness and excess kurtosis of a series are consistent with the Gaussian distribution. Because the level series are $I(1)$, we fit an AR(3) model, $\Delta y_t = c + \rho_1 \Delta y_{t-1} + \rho_2 \Delta y_{t-2} + \rho_3 \Delta y_{t-3} + \varepsilon_t$, to each first-difference series by OLS and apply JB to the residuals. Removing autocorrelation structure before testing the distributional assumption provides a more stringent check of Gaussianity than applying JB directly to the first differences. The JB statistic $\frac{T}{6} [S^2 + \frac{(\kappa-3)^2}{4}]$ is asymptotically $\chi^2(2)$ under the null, where S and κ denote sample skewness and kurtosis of the residuals ($T = 161$ after three-lag alignment). Table 2 reports the results.

Both radiative forcing series reject normality decisively ($\text{JB} > 430$, $p \leq 0.001$), driven by heavy-tailed outliers from major volcanic eruptions (e.g. Krakatoa in 1883, Pinatubo in 1991) that generate large year-on-year swings in forcing not captured by the linear AR(3) dynamics. By contrast, both temperature series are consistent with normality ($p > 0.34$), reflecting the smoothing effect of ocean thermal inertia. This non-normality of radiative forcing innovations invalidates the Gaussian MLE used by bru¹ and provides the key motivation for TAOLS, which does not impose Gaussianity. Histograms and normal Q-Q plots are available upon request.

Table 2 Jarque–Bera Normality Test for AR(3) Residuals of First Differences

Series	Skewness	Excess Kurtosis	JB Stat	<i>p</i> -value
ΔTotalRF	−1.019	8.131	471.3	≤0.001
ΔMarvelRF	−0.893	7.870	436.9	≤0.001
ΔBerkeley	0.104	−0.279	0.8	≥0.500
ΔHadCRUT	0.129	−0.452	1.8	0.341

Notes: AR(3) model $\Delta y_t = c + \rho_1 \Delta y_{t-1} + \rho_2 \Delta y_{t-2} + \rho_3 \Delta y_{t-3} + \varepsilon_t$, fitted by OLS. $T = 161$ residuals (1854–2014). H_0 : normality ($\chi^2(2)$). *p*-values bounded at 0.001/0.500 by MATLAB's `jbttest`.

Multicointegration and ECS

Following bru^1 , we compute ECS from the estimated cointegrating coefficient λ via

$$\text{ECS} = \frac{5.35 \times \ln(2)}{\lambda} = \frac{3.71}{\lambda}, \quad (7)$$

where $3.71 \approx 5.35 \times \ln 2$ is the radiative forcing from a doubling of CO₂ concentrations (equation (6) with $C/C_0 = 2$). Although our empirical model uses aggregate radiative forcing, anthropogenic forcing over the instrumental record is dominated by long-lived greenhouse gases whose aggregate forcing is approximately proportional to CO₂ forcing, so the CO₂-doubling benchmark remains the standard convention.

Our empirical analysis considers three data pairings from bru^1 : Model I uses full-efficacy radiative forcing with Berkeley Earth surface temperature; Model II uses partial-efficacy radiative forcing with Berkeley Earth surface temperature; and Model III uses partial-efficacy radiative forcing with HadCRUT surface temperature. Strictly speaking, these are three data pairings rather than three separate models, but we use the same terminology for easy comparison.

Figures 2–4 display the TAOLS estimates of λ and the implied ECS for each data pairing. In Model I, λ ranges from 1.717 to 2.043 with a mean of 1.991, yielding ECS estimates in $[1.82, 2.16]^\circ\text{C}$ with a mean of 1.87°C . For Model II, λ lies in $[1.488, 1.750]$ with a mean of 1.709, corresponding to ECS in $[2.12, 2.49]^\circ\text{C}$ with a mean of 2.17°C . In Model III, λ ranges from 1.808 to 2.048 with a mean of 2.002, implying ECS in $[1.81, 2.05]^\circ\text{C}$ with a mean of 1.85°C .

These results diverge from those of bru^1 , who employed MLE within an ECM framework. Their MLE estimates of λ are 1.709, 1.326, and 1.567 for Models I, II, and III respectively, implying ECS of 2.17°C , 2.80°C , and 2.37°C . Across all three specifications, the TAOLS estimates of λ exceed the corresponding MLE values, yielding ECS estimates that range from 1.81°C to 2.49°C , all below the corresponding MLE benchmarks. Compared to bru^1 , the standard errors of λ_{TAOLS}

are generally smaller, producing narrower confidence intervals. In each specification, λ increases with K but stabilizes around $K = 80$, beyond which additional basis functions have negligible effect on the estimates, a finding consistent with asymptotic efficiency arguments in SPK¹¹.

A few comments on the choice of K are in order. At the lower end, K has to exceed the number of regressors in the TAOLS multicointegration regression (i.e., 5). Setting $K = 10$ leaves only $10 - 5 = 5$ residual degrees of freedom, the minimum we regard as adequate for reliable variance estimation; reducing to $K = 5$ would exhaust all degrees of freedom ($5 - 5 = 0$), making the residual variance estimator undefined. The asymptotic standard error of each TAOLS coefficient is proportional to $\sqrt{T/K}$; halving K inflates the standard error by a factor of $\sqrt{2}$, which explains the wide confidence bands visible at $K < 30$. At the upper end, K is bounded by the effective sample size after first-differencing (i.e., 164). Moreover, as $K \rightarrow T$, the TAOLS estimator converges to OLS, which suffers from an asymptotic bias that the sine transformation is designed to remove, so we stop short of T at $K = 154$. While a data-driven choice of K based on an asymptotic mean-squared-error criterion exists in principle, it cannot be estimated reliably at sample sizes of this magnitude. More importantly, quantitatively consistent estimates across the entire feasible range of K —as we document here—provide stronger evidence of robustness than any single- K result, which could reflect the particular frequency composition at that specific value of K .

Multicointegration and Heat Feedback

We now examine the estimate of ϕ . Note that ϕ measures the long-run relationship between the heat content of the system and surface temperature. At equilibrium, for a given value of ϕ , a rise in surface temperature by 1°C corresponds to an increase in the system's heat content of approximately ϕ watt-years per square meter ($\text{W}\cdot\text{yr}/\text{m}^2$). For reference, increasing the atmospheric temperature by 1°C requires an in-

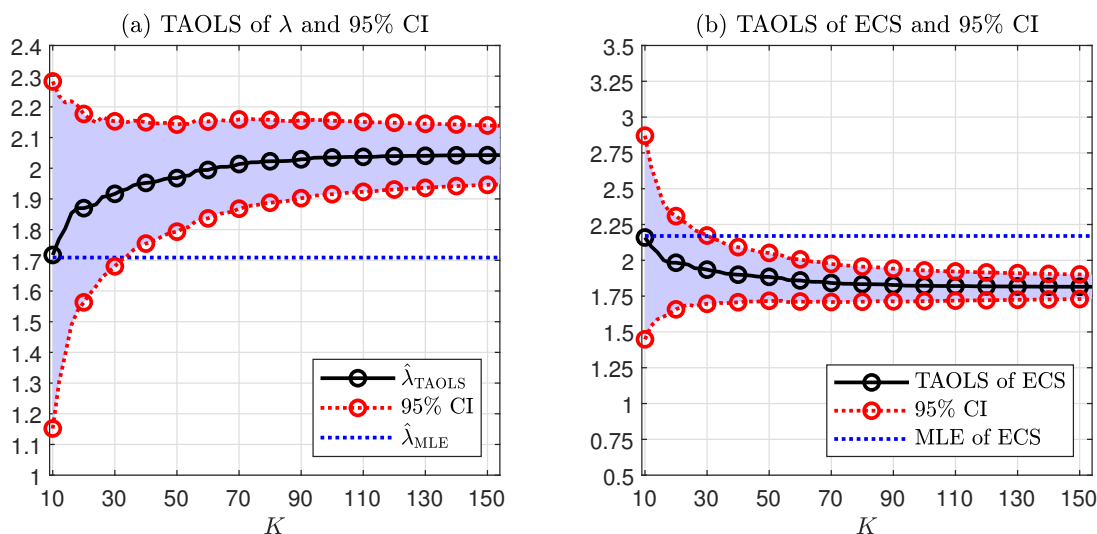


Figure. 2 TAOLS estimate of λ and the implied ECS in the presence of multicointegration for different values of K using full-efficacy radiative forcing and surface temperature from Berkeley Earth (Model I). The shaded region represents the 95% confidence interval.

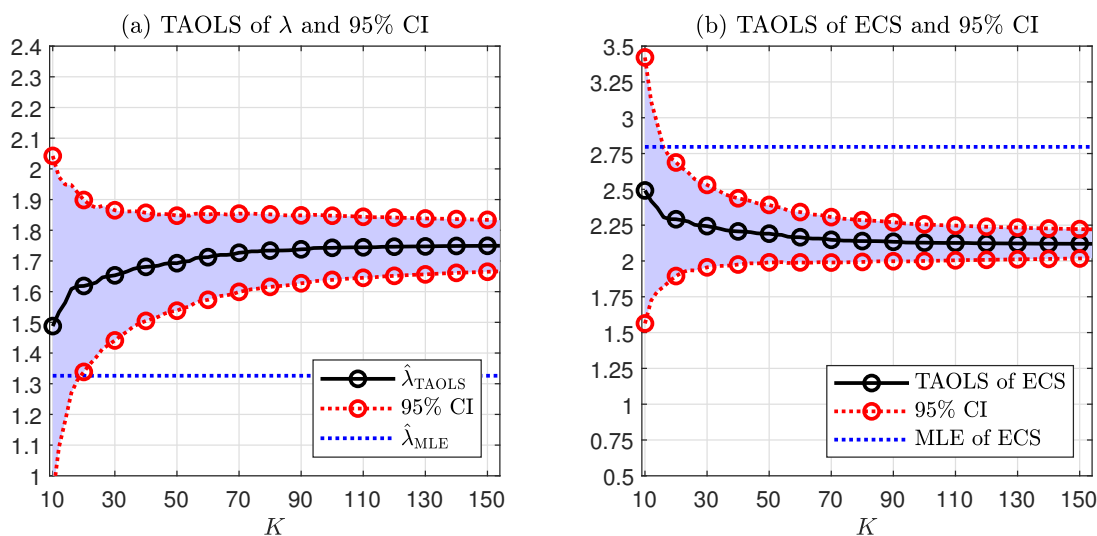


Figure. 3 TAOLS estimate of λ and the implied ECS in the presence of multicointegration for different values of K using partial-efficacy radiative forcing and surface temperature from Berkeley Earth (Model II). The shaded region represents the 95% confidence interval.

crease in heat content of approximately 5×10^{21} joules—derived from the mass of the atmosphere ($\approx 5.15 \times 10^{18}$ kg, inferred from surface pressure measurements) and the specific heat of air at constant pressure ($c_p \approx 1,005 \text{ J kg}^{-1} \text{ K}^{-1}$), giving $5.15 \times 10^{18} \times 1,005 \approx 5.17 \times 10^{21} \text{ J K}^{-1}$ —which translates to 0.31 W-yr/m^2 . Thus, every 0.31 W-yr/m^2 of additional heat absorbed by the atmosphere corresponds to $\phi \text{ W-yr/m}^2$ of the total heat content of the entire system, including the atmosphere, oceans, and other components. This suggests that approximately $31/\phi$ percent of the total heat content contributes to warming the atmosphere.

For Model II, Figure 5 shows that the multicointegration parameter ϕ ranges from 10.41 to 24.51 across the K values. To interpret this physically, consider $\phi = 20$: a 1°C increase in surface temperature corresponds to an additional 20 W-yr/m^2 of stored heat, predominantly in the oceans. This implies that $31/20 \approx 1.5$ percent of the total heat content contributes to atmospheric warming, with the remainder residing in oceanic or terrestrial reservoirs. By comparison, *bru*¹ report ϕ values of 31 to 41, suggesting an air warming fraction of 1% or less. Across all models, the estimated atmospheric share ranges from approximately 0.95% to 2.98%. This range

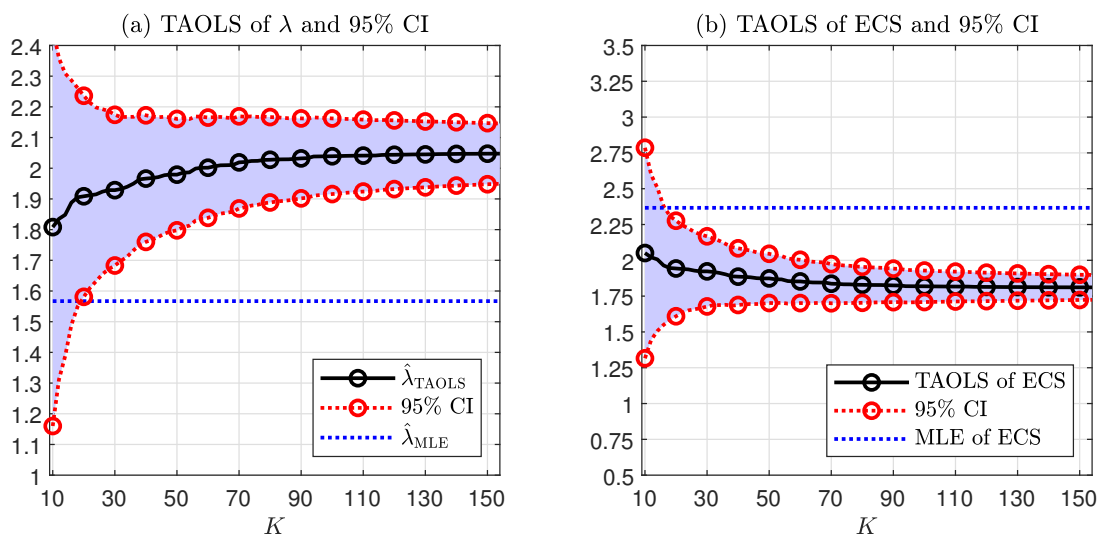


Figure. 4 TAOLS estimate of λ and the implied ECS in the presence of multicointegration for different values of K using partial-efficacy radiative forcing and surface temperature from HadCRUT (Model III). The shaded region represents the 95% confidence interval.

closely matches the empirical estimate of roughly 1% over 1971–2018³ Table 7.1, p. 938. The agreement supports the conclusion that most excess heat is stored in the oceans rather than the atmosphere. Note that $31/\phi$ percent provides an underestimate of the fraction of ocean heat content directed toward surface warming, since the denominator reflects total system heat content rather than the ocean component alone; the difference is small given that the oceans dominate total system heat content.

To illustrate, suppose a sustained radiative forcing increase of 1 W m^{-2} over 10 years accumulates 10 W-yr/m^2 in Q_t . With $\phi = 20$, this heat storage corresponds to a surface temperature increase of approximately $10/20 = 0.5^\circ\text{C}$, illustrating the slow feedback from oceanic reservoirs to atmospheric conditions. This feedback mechanism underscores the multicointegrating structure’s relevance, capturing dynamics beyond the first layer of cointegration.

For Models I and III, the estimates of ϕ are of comparable magnitude to those of Model II. In Model I (full-efficacy radiative forcing, Berkeley Earth), ϕ ranges from 14.69 to 32.74 W-yr/m^2 , implying that approximately 0.95% to 2.11% of the total heat content contributes to warming the atmosphere. In Model III (partial-efficacy radiative forcing, HadCRUT), ϕ ranges from 10.82 to 21.33 W-yr/m^2 , implying approximately 1.45% to 2.87%. These ranges are broadly consistent with those of Model II and reinforce the conclusion that only a small fraction of accumulated heat content drives surface warming, with the remainder residing predominantly in oceanic reservoirs.

Following bru¹, we plot in Figure 6 the TAOLS-predicted system heat content against available ocean heat content

(OHC) observations for Model II (partial-efficacy radiative forcing with Berkeley Earth temperature). The model-implied cumulative Earth system heat content Q_t is converted to units of 10^{22} Joules and scaled to the heat content of the top 2000 m of the ocean by multiplying by 0.81, following bru¹, Section 2.1. The predicted sequence is compared with two observational benchmarks: observed ocean heat content in the top 0–2000 m and 0–700 m from che³⁰ and the simulated series of mar³¹, scaled to the top 2000 m by multiplying by 0.88. The TAOLS-predicted OHC series falls between the 0–700 m series of che³⁰ and the series of mar³¹, tracking the broad upward trend in ocean heat uptake since 1940. Results for Models I and III are quantitatively similar to those shown here.

Conclusions and Discussion

This paper applies the TAOLS estimator of SPK¹¹ to the climate dataset of bru¹, providing the first application of TAOLS to climate data and a direct comparison of the TAOLS and MLE estimates of ECS. TAOLS is more robust than MLE because it does not impose Gaussianity on the model innovations and does not require precise specification of the short-run VAR dynamics—both assumptions that are difficult to maintain in climate time series that exhibit large non-Gaussian spikes from volcanic forcing.

Our ECS estimates from TAOLS, which range from 1.81°C to 2.49°C across all three data pairings, stand in contrast to higher estimates, such as those above 5°C from bjo¹⁴, which emphasize strong positive feedbacks (e.g., cloud dynamics). Our results also exceed the lowest observational estimates, such as the estimate of approximately 1.64°C obtained by

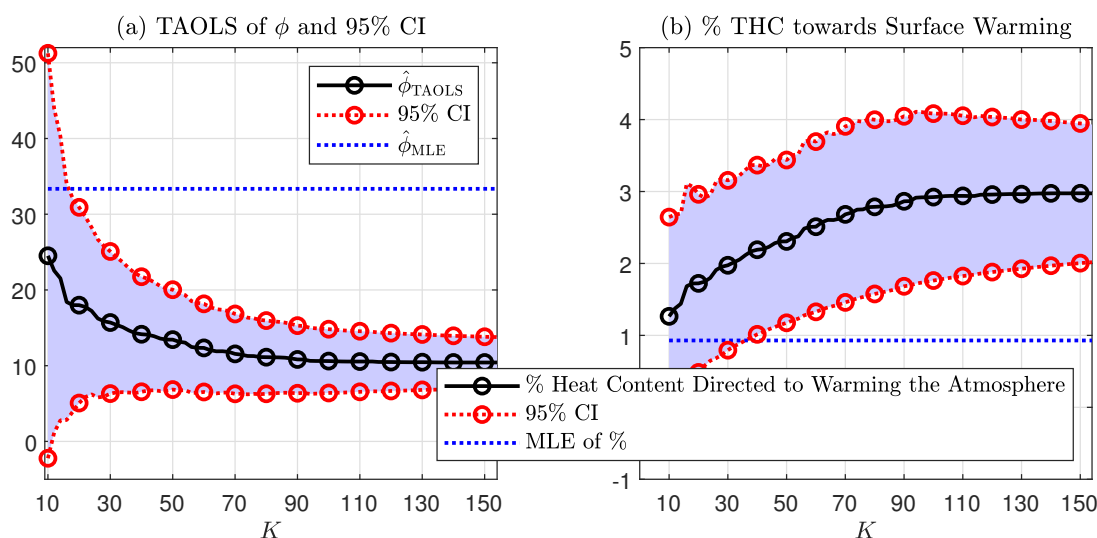


Figure. 5 TAOLS estimate of ϕ and the implied percentage of total heat content directed toward surface warming in the presence of multicointegration for different values of K (Model II). The shaded region represents the 95% confidence interval.

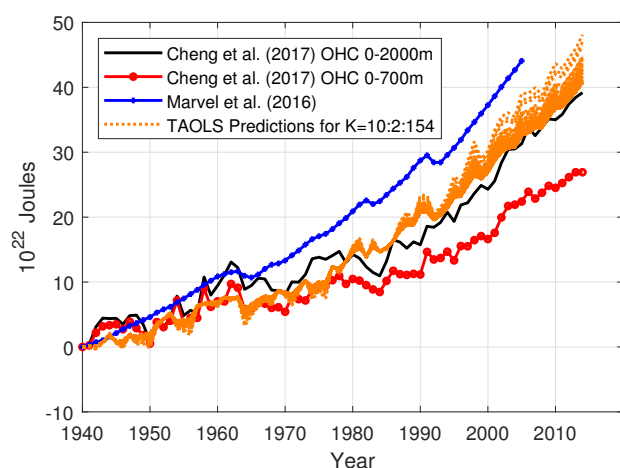


Figure. 6 TAOLS-predicted system heat content compared with observed ocean heat content from che³⁰ (top 0–700 m and 0–2000 m) and simulated heat content from mar³¹ for Model II (partial-efficacy radiative forcing with Berkeley Earth temperature, 1940–2014). The TAOLS prediction (orange) tracks the 0–2000 m series of che³⁰ and falls between the other two series.

low¹³, which some argue reflects the transient rather than the equilibrium response. Compared to the MLE from bru¹, the TAOLS estimates are consistently lower across all three data pairings. We interpret this gap as evidence that the Gaussian VAR model underlying the MLE may be misspecified: when the innovations are non-Gaussian—as they are in climate data with large volcanic spikes—the MLE-based confidence intervals and point estimates can be unreliable, whereas TAOLS remains theoretically valid under weaker distributional assumptions.

tions.

The study has several limitations. The multicointegrating structure is maintained as an assumption, with supporting rank tests drawn directly from bru¹. All relationships are modeled as linear, abstracting from potentially nonlinear climate feedbacks¹⁴. The analysis uses global-mean data, setting aside regional heterogeneity, and the 165-year observational record is short relative to the multi-century equilibration time of the climate system¹. As a reduced-form econometric study, the estimates reflect the statistical properties of the data rather than the output of a mechanistic simulation.

This study suggests that the equilibrium temperature response to a doubling of CO₂ is moderate and remains below 3°C, although this conclusion should be interpreted in light of the study’s limitations. If supported by future work, this result could have implications for climate policy. The analysis also shows that TAOLS can be applied successfully to climate time series. In this application, it produced credible estimates of climate sensitivity with narrower confidence intervals than MLE while avoiding Gaussian distributional assumptions and parametric VAR restrictions. These findings suggest that robust econometric methods may provide a useful alternative for empirical climate research.

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