Optimizing Water Rocket Performance: The Role of Propellant Mass and Launch Angle

Petros Papageorgiou [1]

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Water rockets provide an accessible platform to study fundamental principles of physics, including Newtons laws, momentum conservation, and projectile motion. While prior studies suggest that water mass and launch angle influence flight performance, the quantitative relationship between propellant mass and horizontal range under controlled pressure remains underexplored. This study investigates how water mass and launch angle affect the range of a 1-liter bottle water rocket and identifies the optimal fill level for maximum distance. A custom-built launcher and 1-liter plastic bottle rocket were used to conduct experiments at three launch angles ($45°, 60°, 75°$). Water mass was varied between 100-500g. Each configuration was tested in multiple trials, with horizontal distances measured and averaged. Uncertainties were calculated to account for experimental error. Data were analyzed using projectile kinematics and momentum balance models to predict the dependence of range on propellant mass. A clear quadratic relationship between water mass and horizontal range was observed. Maximum ranges occurred at intermediate water masses of 400-440 g, with measured distances of approximately 26.15 m (45°),$23.25m(60°$), and 15.4 m (75°), closely matching predicted peaks. Distances decreased when water mass exceeded the optimum, reflecting the tradeoff between thrust duration and added weight. Range increased with launch angle across the tested set. The study confirms that bottle rocket performance is optimized at intermediate water fill levels and higher launch angles. These findings reinforce theoretical predictions, inform small-scale rocket design, and provide an accessible model for understanding aerospace principles. Limitations include wind (traveling at 9km/h), launcher alignment, only one kind of rocket design used and a limited trial set.

Keywords: Water rocket, propellant mass, launch angle, horizontal range, projectile motion, experimental physics

Introduction

Rocketry is a central field within aerospace engineering, combining physics, engineering, and applied mathematics to study how rockets generate thrust and overcome atmospheric forces. Although modern spaceflight is highly complex, the fundamental principles of rocketry can be observed using simple experimental models such as water rockets. Water rockets offer a safe and accessible way to investigate how thrust, mass, and aerodynamics influence flight.

Understanding the performance of water rockets requires consideration of Newtons laws of motion, conservation of momentum, and projectile motion. In particular, the amount of water used as propellant plays a critical role: while greater water mass increases propulsion time, it also increases weight, which can reduce launch velocity. Balancing these effects determines the rockets range.

This study investigates the impact of water mass on the horizontal distance travelled by a water rocket and how the horizontal distance depends on the angle of launch. By varying the water mass across trials at different launch angles and analyzing the resulting distances, the experiment aims to identify the relationship between propellant mass and flight performance and the launch angle with the horizontal range travelled. My paper also aims to show the significance of understanding rocket operations. Rocketry is a very crucial part of our history, and it is vital in understanding how our solar system works [1,2]. The universe holds so many unanswered questions and secrets that I am determined to be a small part of this long search.

In this study, I have incorporated physics laws and information from other similar studies, which are properly cited. Due to the availability of resources and time, I was able to construct only one functioning rocket and discuss about three launch angles.

Principle of operations of a water rocket

A simple water rocket has a relatively small opening at one end of its chamber, which allows gas to escape and generates thrust in the opposite direction. This basic operation combines fundamental principles of physics, laws and engineering. More specifically, the second and third laws of Newton [3,4]. are applied.

[1] *Athens College, Greece, Lumiere program*
ppapageorgiou231@athenscollege.edu.gr

As water is pushed outward due to the excess amount of air pressure inside the rocket, according to Newtons third law, an opposite force is exerted on the water rocket. Then, if the vertical component of this force exceeds the weight of the rocket, it will lift off and begin to accelerate in the direction of the net force according to Newtons second law. If the inner pressure has a greater value than the atmospheric pressure and at the same time there is water remaining inside the rocket, it will continue to accelerate. When either the water runs out or the air pressure drops, then the rocket will begin to move under the influence of its weight, accelerating downwards[5]. Different combinations of air pressure and water mass inside the rocket will provide different amounts of thrust, affecting its range.

To further analyze the case of a water rocket, when mass is ejected from the rocket, a forward opposed force is applied to the rocket itself. By applying the principle of conservation of momentum, the propulsion force ΣF can be derived. This force is equal to the rate of ejection of mass multiplied by the relative ejection speed of the mass to the rocket. This derivation can be done by considering the conservation of mass of the rocket and the mass of the ejected gases as a function of the conservation of momentum.

$$M \cdot v = M - dm \cdot (v + dv) - dm \cdot u - v - dv$$

$$M \cdot v = M \cdot v + M \cdot dv - dm \cdot v - dm \cdot dv - dm \cdot u + dm \cdot v - dm \cdot dv$$

$$0 = M \cdot dv - dm \cdot u \rightarrow M \cdot dv = dm \cdot u$$

$$M \cdot \frac{dv}{dt} = \frac{dm}{dt} \cdot u \rightarrow M \cdot a = \frac{dm}{dt} \cdot u \rightarrow \Sigma F = \frac{dm}{dt} \cdot u \,(\text{eq } 1)$$

Hence, using the force calculated in the previous step, the Impulse delivered during

propulsion time is:

$$I = \Sigma F \cdot t \,(\text{eq } 2)$$

If I denote as $m(t)$ the function of the mass of the water rocket (changing with time), then combining the two equations, the launch speed v is:

$$I = \frac{dm}{dt} \cdot u \cdot t \rightarrow m(t) \cdot v - m(t) \cdot v_0 = \frac{dm}{dt} \cdot u \cdot t \rightarrow v = \frac{\frac{dm}{dt} \cdot u \cdot t}{m(t)} \text{ eq } 3$$

The latter equation reveals that if the mass ejection rate $\frac{dm}{dt}$ and the velocity u of the expelled water are constant, then the final velocity v of the rocket is inversely proportional to the total mass of the rocket. Thus, adding more water increases the thrust delivered, but the increased mass simultaneously harms the launch. The range of the water rocket system can be predicted by the kinematic equations of a projectile motion at a launch angle to the ground, neglecting air resistance. Specifically, the net vertical displacement of the rocket, from launch to landing, is 0.

$$\Delta y = 0 \rightarrow v_y \cdot t - \frac{1}{2} \cdot g \cdot t^2 = 0 \rightarrow t = \frac{2v_y}{g} \,(eq4)$$

$$\Delta y = 0 \rightarrow v_y t - \frac{1}{2} g t^2 = 0 \rightarrow t = \frac{2v_y}{g} \text{ (eq 4)}$$

Thus, in the x-axis, the range could be calculated from the equation below,

$$x = v_x t \rightarrow x = v_x \cdot \frac{2v_y}{g} \rightarrow v \cos\theta \cdot \frac{2v \sin\theta}{g} \rightarrow x = \frac{2v^2 \cos\theta \sin\theta}{g} \rightarrow$$

$$x = \frac{\sin 2\theta \cdot v^2}{g} \rightarrow x = \frac{\sin 2\theta}{g} \cdot \left(\frac{\frac{dm}{dt} \cdot u \cdot t}{m(t)} \right)^2 \quad (eq.5)$$

This equation suggests a quadratic relationship between the range travelled and the mass inside a water rocket. This hypothesis is not only supported by the above equation; however, Studies by Pathan[6] and Acciani[7] also hypothesized a quadratic relationship between launch mass and rocket range under similar experimental conditions.

Methods

This is an experimental research paper, containing field work and the actual construction of a water rocket.

Tools needed

Table 1 shows all the materials and tools I used to build the launcher and the rocket.

DJ set
Case of a saw tool
Plastic egg case
Metal pipe with a big opening
Metal pipe with small opening
Wine cork
Air pump
Duct tape
Measuring tape
Protractor
Water bottle 1L
Plastic for fins
Tennis ball
Measuring tape
Weighing scale
Tap water

Table 1 Tools needed for the experiment

Construction of the launcher

The launcher was constructed in two main parts. For the first part of the launcher, a DJ stand available on-site was used as the base, allowing adjustment of both the launch angle and height. The casing of a saw tool, which was elongated, was attached to serve as the takeoff ramp. This component was secured using duct tape. Additionally, a small piece was cut from a plastic egg carton and positioned to act as a stopper, preventing the rocket from sliding downwards while the launcher was inclined as seen in Figure 1.

For the second part of the launcher, we used an electric pump to generate the air pressure inside the rocket. We connected a white metal pipe with two large openings at its ends to the pump so that the air could flow into the rocket. At the opposite end of the pipe, we used a wine bottle cork to seal the rocket, trapping both the air and water while also connecting the rocket to the pipe. Finally, we drilled a hole with diameter and tightly

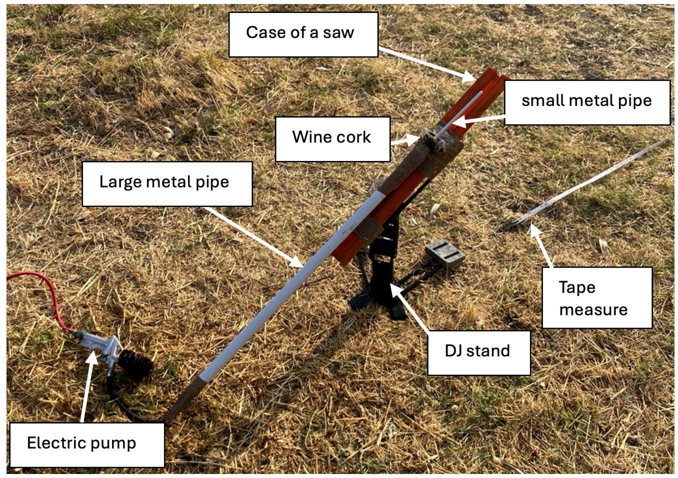

Fig. 1 Rocket launcher

inserted a smaller pipe with a very narrow opening through the cork and into the rocket. This smaller pipe allowed air to enter the rocket while preventing water from escaping.

Construction of the water rocket

A plastic bottle was used as the main body of the water rocket. Three plastic fins were attached to the sides of the bottle with duct tape to provide stability during flight. A tennis ball was secured to the top of the bottle to serve as a nose cone, improving stability and reducing air resistance. Duct tape was also applied to reinforce weak points in the rockets structure. The rocket was pressurized with water and air using a pump in order to generate thrust for launch.

Experimental procedure

The experimental procedure was carried out according to the following steps:

1. We measured specific masses of water, ranging from (100 - 500)g, determining their mass using a weighing scale with a known uncertainty of $\pm g$.
2. The water was poured into the empty plastic bottle.
3. Then the cork was tightly lodged into the opening of the water rocket until a specific mark at the middle of the length of the cork, to prevent air or water from escaping.
4. We used a pressure pump to inject air into the bottle until the pressure of about $\overline{(3.2 \pm 0.2)}$ forced the rocket to launch.
5. After each launch, we measured the horizontal distance travelled, using a measuring tape.

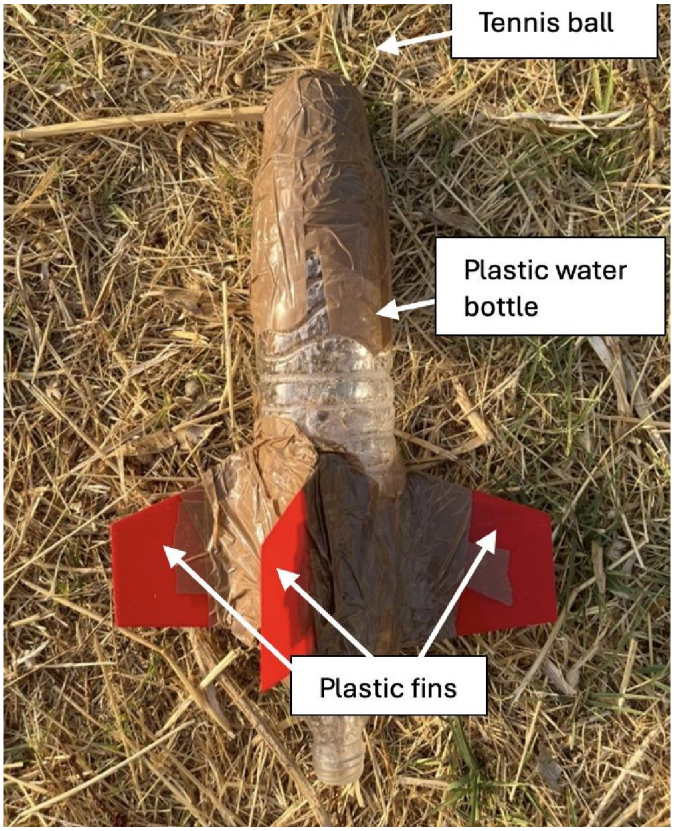

Fig. 2 Water rocket

Fig. 3 Gauge meter used

6. We repeated the experiment at different launch angles, using a protractor to set and record each angle of launch paying special attention inserting the cork to the specific marked point at every trial.

7. All data, including distance travelled (m), air pressure (bar), water mass (g), and angle (degrees), was recorded in an Excel spreadsheet for analysis.

The above methodology and construction of both the rocket and the launcher were used to ensure efficiency and safety in the experiment. More specifically, I set the rocket to travel in the x-axis and not in the y-axis as I couldnt find an altimeter that satisfied the needs of the experiment. Therefore, to have reliable data I measured the range which was feasible with the resources I had available. In addition, by measuring only the range traveled, the experiment involved minimal risk, as the rocket was launched away from me, ensuring no possible harmful outcome. The rocket design was similar to other studies that conducted experiments with water rockets and was obviously a very small and simple representation of a regular rocket as in general most space rockets have a few fins (mostly 3) and a nose cone to minimize air resistance.

Prior to initiating the measuring process I did some trials to determine the pressure at which the cork would pop out from the rocket as measured by the gauge meter depicted in Figure 3. This instrument measures the amount of pressure beyond the atmospheric pressure, that is why it is refereed as gauge pressure. Moreover, I wanted to verify that the cork would burst out at roughly the same pressure which was measured to be The uncertainty of the launch pressure exceeds the reading uncertainty of the gauge meter intending to consider the fact that the launch was triggered at approximately without the use of a more specific launch mechanism such a as a pressure valve.

Variables

As shown in Table 2, these were the variables that were considered before and after the experiment and the analysis of the data.

Variables	Ways of measuring/controlling them
Controlled: Same initial height.	We measured the initial height of each launch using a tape measure and made sure to never adjust it and keep it at the same value.
Controlled: Same rocket shape.	The rocket was the same (same water bottle 1L) throughout the experiment. This ensured the same aerodynamics and that the range was not affected by different shapes.
Controlled: Same diameter of bottle opening.	By using the same rocket, we also ensured that the exit velocity depended only on the internal pressure and water volume, since the same forces were applied to the rocket through the experiment.
Controlled: Same launch pressure.	Throughout the experiment the same electric pump was used. The pressure was kept approximately the same. If the pressure varied, then the range of the rocket would also be affected.
Controlled: Same source of water.	We used the same source of water for each launch (tap water). Different densities or impurities could result in different volumes and thus affect each result.
Controlled: Wind speed.	We made sure to take measurements on the same evening. That is, we launched when the air was not very windy, so that the air was travelling at a speed of 9 km/h with approximately constant direction.
Controlled: No deviation in the measuring tape.	The measuring tape reached 25 m. It was placed right underneath the launch line to ensure that the correct path of the bottle would not have been recorded otherwise.
Independent: Angle of launch to the horizontal	We changed the launch angle of the rocket to see how it affected the distance travelled. The angles tested were 45° , 60° , and 75° . We measured the angle using a protractor.
Independent: Mass of rocket	We measured the initial mass of the rocket which was 420 g. Then we added water and recorded the masses of water that were added to the rocket, with an uncertainty of ±20 g.

Table 2 Variables of the experiment

Angle φ $^\circ/\pm0.5^\circ$	Mass of water m_1 $g/\pm1g$	Mass of water m_2 $g/\pm1g$	Air pressure bar$/\pm0.2$	Distance travelled d_1/m_1	Distance travelled d_2/m_2
45	101	100	3.2	12.8	13.5
45	202	203	3.2	17.8	18.4
45	304	299	3.2	23.3	24.1
45	402	400	3.2	26.4	25.9
45	501	505	3.2	24.1	24.5

Table 3 Raw data for launch at an angle of 45 degrees

Angle φ $^\circ/\pm0.5^\circ$	Mass of water m_1 $g/\pm1g$	Mass of water m_2 $g/\pm1g$	Air pressure bar$/\pm0.2$	Distance travelled d_1/m_1	Distance travelled d_2/m_2
60	101	103	3.2	12.8	12.3
60	202	200	3.2	17.9	15.4
60	303	301	3.2	21.3	18.1
60	402	404	3.2	22.9	23.6
60	505	501	3.2	21.4	21.6

Table 4 Raw data for launch at an angle of 60 degrees

Angle φ $^\circ/\pm0.5^\circ$	Mass of water m_1 $g/\pm1g$	Mass of water m_2 $g/\pm1g$	Air pressure bar$/\pm0.2$	Distance travelled d_1/m_1	Distance travelled d_2/m_2
75	102	104	3.2	9.5	9.1
75	202	201	3.2	12.1	11.7
75	304	305	3.2	12.6	13.3
75	402	401	3.2	15.7	15.1
75	506	502	3.2	13.9	14.7

Table 5 Raw data for launch at an angle of 75 degrees

Angle φ $^\circ/\pm0.5^\circ$	Average mass of water m_{av}/g	Δm_{av}/g	Air pressure bar$/\pm0.2$	Distance travelled d_{av}/m	Δd_{av}/m
45	100.5	0.5	3.2	13.2	0.4
45	202.5	0.5	3.2	18.1	0.3
45	301.5	2.5	3.2	23.7	0.4
45	401.0	1.0	3.2	26.2	0.3
45	503	2.0	3.2	24.3	0.2

Table 6 Processed data for launch at an angle of 45 degrees

Results

Raw data

The tables 3 4 5 show the raw data collected during the launches for angles of 45,60,75 degrees respectively.

Processed data

For the processed data I will calculate the average distance as well as the uncertainty,

$$d_{\text{avg}} = \frac{d_1 + d_2}{2}$$

$$d_{\text{avg}} = \frac{12.8 + 13.5}{2}$$

$$d_{\text{avg}} = 13.2 \text{ m}$$

To calculate the uncertainty of the distance,

$$\Delta d_{\text{avg}} = \frac{d_{\text{max}} - d_{\text{min}}}{2}$$

$$\Delta d_{\text{avg}} = \frac{13.5 - 12.8}{2}$$

$$\Delta d_{\text{avg}} = \pm 0.4 \text{ m}$$

$$d_{\text{avg}} = 13.2 \pm 0.4 \text{ m}$$

Similarly, for the mass of water inside the rocket,

$$m_{\text{avg}} = \frac{m_1 + m_2}{2}$$

$$m_{\text{avg}} = \frac{101 + 100}{2}$$

$$m_{\text{avg}} = 100.5 \text{ g}$$

$$\Delta m_{\text{avg}} = \frac{m_{\text{max}} - m_{\text{min}}}{2}$$

$$\Delta m_{\text{avg}} = \frac{101 - 100}{2}$$

$$\Delta m_{\text{avg}} = \pm 0.5 \text{ g}$$

$$m_{\text{avg}} = 100.5 \pm 0.5 \text{ g}$$

Table 6 presents the analyzed data for an angle of 45 degrees.

For the first series of launches, at 45 degrees, the curve fit had the highest correlation coefficient of 0.9888. This indicates the presence of small errors during the experiment. For example, air resistance played a partial role towards the errors as on the day of launch the wind speed was measured at at an opposite direction to the launches (based on meteo.gr), which of course could not have been steady throughout the measuring process as there were slight changes to the wind speed. Moreover, during the launch the rocket could have taken a slightly different trajectory during each different measurement which also affects

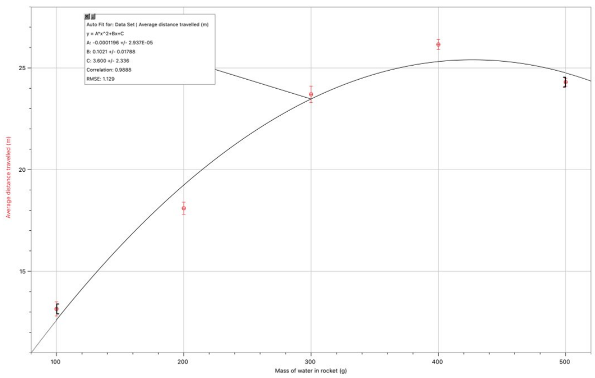

Fig. 4 Average distance (m) vs water mass (g) for 45 degrees

the distance travelled. Lastly, the mass of the water inside the rocket wasnt always exactly the same during both launches (i.e. for the the first time it was and the second), this also affected the data points slightly. The error bars that appear in graph 1 express the half-range method of measured quantities.

In addition, a very useful statistical parameter is . This tells you the proportion of the variance in the dependent variable that is explained by the independent variable(s) in a regression model. It measures the goodness of fit of the model to the observed data, indicating how well the model's predictions match the actual data points[8]. Satisfactory values of depend on the context of both the subject and the experiment conducted, thus in this specific experiment the values indicate strong fits from . To calculate the value of for a specific graph I applied the formula

$$R^2 = 1 - \frac{\sum \left(d_{\text{avg},i} - \overline{d}'_{\text{avg}}\right)^2}{\sum \left(d_{\text{avg},i} - \overline{d}_{\text{avg}}\right)^2} = 0.978$$

Where $\overline{d}_{\text{avg}}$ is the mean value of the horizontal distance travelled and $\overline{d}'_{\text{avg}}$ the predicted value from the regression model.

The last metric that can be obtained from the values and the graph is RMSE, which is a standard statistical metric used to evaluate the performance of predictive model. In this case, the RMSE has a value of **1.129** that shows the validity of the data and also supports the claim that there are still improvements that can be made to reduce the uncertainties of the measurements.

To find the projected maximum distance that could be travelled at that angle I will use the formula for the vertex of a quadratic graph. More specifically, the graph is of the form

$$y = Ax^2 + Bx + C,$$

where

$$A = (-12 \pm 3) \cdot 10^{-5}\ \text{m} \cdot \text{g}^{-2}, B = (0.10 \pm 0.02)\ \text{m} \cdot \text{g}^{-1}, C = (4 \pm 2)\ \text{m}.$$

$$x_{\max} = \frac{-B}{2 \cdot A}$$

$$x_{\max} = \frac{0.1}{2 \cdot (-12 \cdot 10^{-5})}$$

$$x_{\max} = 416.7\ \text{g}$$

For the uncertainty,

$$\frac{\delta x_{\max}}{x_{\max}} = \frac{\delta A}{A} + \frac{\delta B}{B}$$

$$\delta x_{\max} = x_{\max} \cdot \left(\frac{\delta A}{A} + \frac{\delta B}{B}\right)$$

$$\delta x_{\max} = 416.7 \cdot \left(\frac{3}{12} + \frac{0.02}{0.10}\right)$$

$$\delta x_{\max} = 20$$

$$x_{\max} = 420 \pm 20\ \text{g}$$

Therefore,

To find the maximum average distance *travelled* I will use the above value found in the equation of the best fit curve,

$$y_{\max} = (-12 \cdot 10^{-5}) \cdot x_{\max}^2 + 0.10 \cdot x_{\max} + 4$$

$$y_{\max} = (-12 \cdot 10^{-5}) \cdot 420^2 + 0.10 \cdot 420 + 4$$

$$y_{\max} = 24.8\ \text{m}$$

However, to find the uncertainty of the maximum value of y, I must use partial derivatives:

Firstly, the equation is:

$$y = A x_{\max}^2 + B x_{\max} + C$$

$$y = A\left(\frac{B}{4A^2}\right) - B\left(\frac{B}{2A}\right) + C$$

$$y = \frac{B^2}{4A} - \frac{B^2}{2A} + C$$

$$y = -\frac{B^2}{4A} + C$$

Then to find its uncertainty,

$$\delta y = \sqrt{\left(\frac{\partial y}{\partial B} \cdot \delta B\right)^2 + \left(\frac{\partial y}{\partial A} \cdot \delta A\right)^2 + \left(\frac{\partial y}{\partial C} \cdot \delta C\right)^2}$$

In this case:

$$\frac{\partial y}{\partial B} = \frac{-2B}{A}, \qquad \frac{\partial y}{\partial A} = \frac{B^2}{4A^2}, \qquad \frac{\partial y}{\partial C} = 1$$

Angle φ $^\circ/\pm0.5^\circ$	Average mass of water m_{av}/g	Δm_{av}/g	Air pressure bar/±0.2	Distance travelled d_{av}/m	Δd_{av}/m
60	102.0	1.0	3.2	12.55	0.25
60	201.0	1.0	3.2	16.65	1.25
60	302.0	1.0	3.2	19.7	1.6
60	403.0	1.0	3.2	23.25	0.35
60	503	2.0	3.2	21.5	0.1

Table 7 Processed data for launch at an angle of 60 degrees

Therefore, the uncertainty of y becomes

$$\delta y = \sqrt{\left(\frac{-2B}{A}\cdot\delta B\right)^2 + \left(\frac{B^2}{4A^2}\cdot\delta A\right)^2 + (1\cdot\delta C)^2}$$

$$\delta y = \sqrt{\left(\frac{-2\cdot 0.10}{-12}\cdot 0.02\right)^2 + \left(\frac{0.10^2}{4\cdot(-12)^2}\cdot 3\right)^2 + (1\cdot 2)^2}$$

$$\delta y = 2 \text{ m}$$

Thus,

$$y = 25 \pm 2 \text{ m}$$

The curve fit suggests that to reach a maximum distance at an angle of 45 degrees, I must use 420 grams of water with an associated margin of error of 20 grams. This would make the rocket reach a distance of (25 ± 2) meters. Therefore, not only does graph 1 has a high correlation, which indicates that the launches and measurements were efficient and accurate, it is also in accordance with equation 5 which suggests that at an angle of 45 degrees the water rocket would be able to travel the highest maximum distance.

The presence of the positive y-intercept implies that even without any water mass inside the rocket, the pressurized air could lead to a range of (4 ± 2) m for the empty bottle at a launch angle of 45 degrees, which of course is something that could be tested in case of a repeating the measuring process.

Table 7 presents the analyzed data for an angle of 60 degrees.

For the launch at 60 degrees the correlation of the graph is 0.9818 which indicates a good fit. This series of launches had mostly the same errors during the experiment as the first series of launches at 45 degrees. By using the same process, the graph is of the form $y = Ax^2 + Bx + C$, where $A = (-8\pm3)\cdot10^{-3}$ m $\cdot$ g^{-2}, $B = (0.07\pm0.02)$ m $\cdot$ g^{-1}, and $C = (6\pm2)$ m.

Therefore, to calculate the value of R^2 for this graph,

$$R^2 = 1 - \frac{\sum\left(d_{\text{avg},i} - \overline{d}'_{\text{avg}}\right)^2}{\sum\left(d_{\text{avg},i} - \overline{d}_{\text{avg}}\right)^2} = 0.984$$

To find the maximum distance traveled suggested by the graph I will use the vertex again:

$$x_{\max} = \frac{-B}{2\cdot A}$$

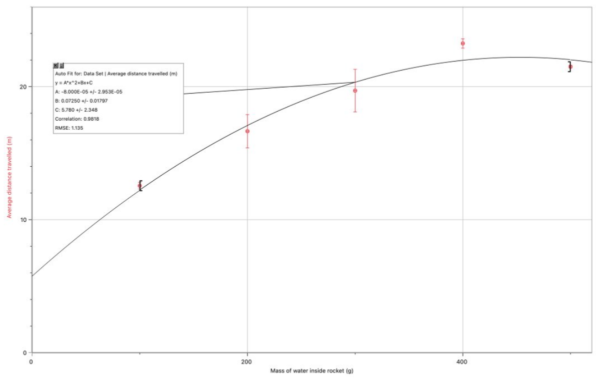

Fig. 5 Average distance (m) vs water mass (g) for 60 degrees

$$x_{\max} = \frac{0.07}{2\cdot(-8\cdot10^{-3})}$$

$$x_{\max} = 437.5 \text{ g}$$

For the uncertainty,

$$\frac{\delta x_{\max}}{x_{\max}} = \frac{\delta A}{A} + \frac{\delta B}{B}$$

$$\delta x_{\max} = x_{\max}\cdot\left(\frac{\delta A}{A} + \frac{\delta B}{B}\right)$$

$$\delta x_{\max} = 437.5\cdot\left(\frac{3}{8} + \frac{0.02}{0.07}\right)$$

$$\delta x_{\max} = 40 \text{ g}$$

$$x_{\max} = 440 \pm 40 \text{ g}$$

To find the maximum average distance *travelled* I will use the above value found in the equation of the best fit curve,

$$y_{\max} = (-8\cdot10^{-3})\cdot(440)^2 + 0.07\cdot(440) + 6$$

$$y_{\max} = 21.3 \text{ m}$$

For the uncertainty,

$$\delta y = \sqrt{\left(\frac{2B}{A}\cdot\delta B\right)^2 + \left(\frac{B^2}{4A^2}\cdot\delta A\right)^2 + (1\cdot\delta C)^2}$$

$$\delta y = \sqrt{\left(\frac{2\cdot 0.07}{-8\cdot10^{-3}}\cdot 0.02\right)^2 + \left(\frac{(0.07)^2}{4\cdot(-8\cdot10^{-3})^2}\cdot 3\right)^2 + (1\cdot 2)^2}$$

$$\delta y = 2 \text{ m}$$

$$y = 21 \pm 2 \text{ m}$$

Angle φ $°/\pm0.5°$	Average mass of water m_{av}/g	Δm_{av}/g	Air pressure bar/±0.2	Distance travelled d_{av}/m	Δd_{av}/m
75	103.0	1.0	3.2	9.3	0.2
75	201.5	0.5	3.2	17.9	0.2
75	304.5	0.5	3.2	13.0	0.4
75	401.5	0.5	3.2	15.4	0.3
75	504	2.0	3.2	14.3	0.4

Table 8 Processed data for launch at an angle of 75 degrees

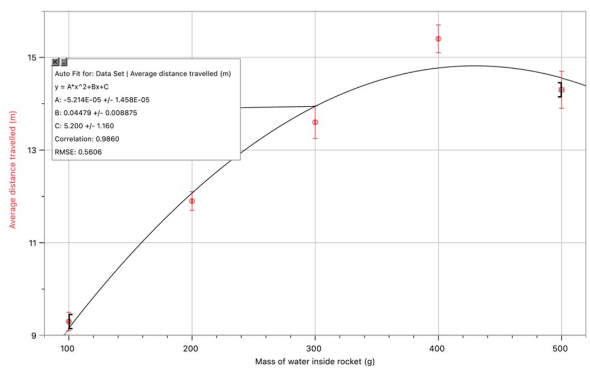

Fig. 6 Average distance (m) vs water mass (g) for 75 degrees

The curve fit suggests that to reach a maximum distance at an angle of 60 degrees, I must use 440 grams of water with an associated margin of error of 40 grams. This would make the rocket reach a distance of 21 meters with an associated margin of error of 2 meters. Therefore, not only does graph 2 has a high correlation, which indicates that the launches and measurements were efficient and accurate, it is also in accordance with equation 5 which suggests that at an angle of 60 degrees the water rocket would be able to travel the highest maximum distance.

The presence of the positive y-intercept implies that even without any water mass inside the rocket, the pressurized air could lead to a range of 6 ± 2 m for the empty bottle at a launch agle of 60 degrees, which of course is something that could be tested in case of a repeting the measuring process.

Table 8 presents the analyzed data for an angle of 75 degrees.

For the launch at 75 degrees, the graphs correlation coefficient is 0.9860 which indicates both the success of the launches but the presence of small errors or factors affecting the experiment.

Therefore, to calculate the value of R^2 for graph 3,

$$R^2 = 1 - \frac{\sum \left(d_{\text{avg},i} - \overline{d}'_{\text{avg}}\right)^2}{\sum \left(d_{\text{avg},i} - \overline{d}_{\text{avg}}\right)^2} = 0.942$$

While the value of R^2 for graph 3 is lower than the two previous graphs, the value of RMSE is much higher at 0.5066 which strongly signifies a better model accuracy than the two previous launch angles.

As seen from the other two graphs, the quadratic equation which links the average distance travelled d_{av} with the mass of the water inserted in the rocket m is of the form

$$y = Ax^2 + Bx + C,$$

where

$$A = (-5.2 \pm 1.5) \cdot 10^{-5} \text{ m} \cdot \text{g}^{-2},$$
$$B = (0.045 \pm 0.008) \text{ m} \cdot \text{g}^{-1},$$
$$C = 5.2 \pm 1.2 \text{ m}.$$

Thus, with the above data, the best fit curve states that the maximum distance is the following maximum y-coordinate and it can be reached by using the following mass of water inside the rocket:

$$x_{\max} = \frac{-B}{2 \cdot A}$$

$$x_{\max} = \frac{0.045}{2 \cdot (-5.2 \cdot 10^{-5})}$$

$$x_{\max} = 432.7 \text{ g}$$

For the uncertainty,

$$\frac{\delta x_{\max}}{x_{\max}} = \frac{\delta A}{A} + \frac{\delta B}{B}$$

$$\delta x_{\max} = x_{\max} \cdot \left(\frac{\delta A}{A} + \frac{\delta B}{B}\right)$$

$$\delta x_{\max} = 432.7 \cdot \left(\frac{1.5}{5.2} + \frac{0.008}{0.045}\right)$$

$$\delta x_{\max} = 50 \text{ g}$$

Therefore,

$$x_{\max} = 430 \pm 50 \text{ g}$$

To find the maximum average distance *travelled* I will use the above value found in the equation of the best fit curve,

$$y_{\max} = (-5.2 \cdot 10^{-5}) \cdot x_{\max}^2 + 0.045 \cdot x_{\max} + 5.2$$

$$y_{\max} = (-5.2 \cdot 10^{-5}) \cdot 430^2 + 0.045 \cdot 430 + 5.2$$

$$y_{\max} = 14.9 \text{ m}$$

To calculate its uncertainty as illustrated before,

$$\delta y = \sqrt{\left(\frac{-2B}{A} \cdot \delta B\right)^2 + \left(\frac{B^2}{4A^2} \cdot \delta A\right)^2 + (1 \cdot \delta C)^2}$$

$$\delta y = \sqrt{\left(\frac{-2 \cdot 0.045}{-5.2} \cdot 0.008\right)^2 + \left(\frac{0.045^2}{4 \cdot (-5.2)^2} \cdot 1.5\right)^2 + (1 \cdot 1.2)^2}$$

$$\delta y = 1.2 \text{ m}$$

The vertex of the above graph is Vertex $(430 \pm 50,\ 14.9 \pm 1.2)$. The curve fit indicates that the maximum distance can be accomplished using (430 ± 50) g of water. In comparison to my results, in which we achieved a very similar maximum distance, of 15.4 m, using approximately 400 g of water.

The presence of the positive y-intercept implies that even without any water mass inside the rocket, the pressurized air could lead to a range of (5.1 ± 1.2) m for the empty bottle at a launch agle of 75 degrees, which of course is something that could be tested in case of a repeting the measuring process.

Therefore, it is seen that as the angle of launch increases the distance travelled decreases which can be also verified from equation 5 as the maximum value for $\sin 2\theta$ can be obtained at 45 degrees which is equal to 1.

Discussion

Conclusions and future work

In conclusion, based on the data analysis, the equation that relates the distance travelled by the water rocket to mass is of the form $y = Ax^2 + bx + c$.

The recorded data indicate that for water masses between 100–440 grams, the distance travelled increases. However, beyond approximately 440 grams, the distance travelled began to decrease. This observation is not only corroborated by the experimental data and best-fit curves, but also by a Spearman's rank correlation analysis which had a high value in all three graphs (0,9888;0,9818;0,9860)[9], which confirmed a statistically significant positive correlation between mass and range within the 100-440 g interval. When masses exceeded this threshold, a notable decline in distance was observed.

When observing the graphs, some data points, along with their error bars, did not align precisely with the best fit curve. While this overall trend is supported by both my results and prior research[10] a clearer and more precise conclusion would require additional repetitions and a broader range of mass values, something that was not feasible within the time and resource constraints of the research program. Moreover, the difference in the uncertainties of the data points shows there were factors that, regardless of simple human errors, affected the launches and the ranges measured. To be precise, the wind resistance during the launch (opposing the launch of the rocket at a speed of 9km/h), the friction of the ramp, very small changes in the shape of the rocket as it was used many times and differences in the mass used across measurements are all factors which affect both the trajectory of the rocket and the distance that it travels. This is also visible in the uncertainties at the best fit curve created by logger pro. In all three graphs, the factors A, B, C all have a related uncertainty due to the above errors.

The experimental findings are consistent with previous research. To begin with, as mentioned in the section of the principles of operation of a water rocket, the two studies, Pathan[6,7], that hypothesized a quadratic relationship between the two studied variables also reached the same conclusion after conducting research or an experiment like mine. Furthermore, Toma[11–13] found that optimal performance occurs when the applied pressure is approximately 7 atm and the water volume represents 40-50% of the rockets capacity. Additional studies provide further support: Yang[14] demonstrated that excess water reduces range due to added weight, while insufficient water results in inadequate thrust[15–17]. Romrell[18] confirmed experimentally that maximum distance is achieved with an optimal water mass that balances thrust generation with reduced weight resistance.

Overall, the conclusion reached is that the relationship between launch mass and distance travelled is quadratic in nature. This outcome is not only evident in the collected data but is also supported by the underlying physics of thrust and momentum, as well as by multiple independent studies[10,19,20].

Future improvements to the experiment should focus on expanding the dataset. A more extensive study could include additional launch angles (at least to determine the optimal launch trajectory[21–23] and a measurement of the intial speed via video-based velocimetry through programs such as Capestone. In addition, a wider range of water masses could be used or rockets of varying shapes to assess aerodynamic performance. Moreover, for more precise results the launcher could be improved by using guide rails for repeatable aim and different nozzle diameters[24] to see the dependence of the horizontal range travelled on the opening of the nozzel. By launching rockets at different wind speeds while keeping mass and angle constant, it would be possible to quantify the influence of air resistance on rocket range. Furthermore, another promising avenue of investigation would be to systematically evaluate the role of wind resistance. In other words, to adress wind issues, the experiment could be carried out indoors, or wind limits could be set prior to the launches. Another way of adressing the problem of wind speed is by connecting a wind meter to a laptop and record the wind speed variation during the measurements. This would allow me to use the exact value of the wind speed and consider them it into my measurements. In addition, to show how the pressure was valued during the launches, record videos of the gauge meter and create a pressure vs time graph[25,26] to map thrust phase parameters. Lastly, record videos of the propulsion of the water rocket to record the initial velocity and analyze its launch[20,27,28]. These extensions would strengthen the conclusions of this study and contribute to a more comprehensive understanding of water rocket dynamics.

Concerns and limitations

In table 9, we are analyzing the concerns and limitations we faced in the experiment, including where the rocket was pointed, the wind speed during the launch, the range of masses used, the number of trials conducted, the shape of the rocket and the available equipment.

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Error in the experiment	Small analysis of the error	Way of improving
Aiming of the rocket	Unfortunately, due to errors, human and in the setup, the rocket wasnt pointed in the same direction in each launch. There were slight differences that occurred during the launches which affected the distance travelled,	We could attach a small rail system on the ramp of the launcher that holds the rocket in the same place throughout the launch and doesnt change at every launch.
Wind speed and its direction	The wind present at the experiment was another restriction that the rocket had to overcome. The wind affected the distance reached. When analyzing the data I didnt consider the wind speed (5 m/s) as a factor, which is a limitation of the experiment.	To improve or use the wind speed towards the rockets advantage, we would have to point the rocket at the same way that the air travels to use a possible restriction as another thrust force. This, if achieved, could help the rocket travel further. However, it is very difficult to achieve this as wind direction changes spontaneously. Therefore, to reduce this restriction, the measurements would have to be taken in an indoor place.
Range of masses	The masses that we used ranged from 100-500 grams. Based on theory and research, masses from below 500 grams would follow the same pattern, however we couldnt come to that conclusion without conducting an experiment.	To improve this limitation, a wider range of masses could have been used. More specifically, using masses ranging from 100-1000 grams or using either 100 grams or 50 grams intervals to reach a better conclusion about the effective launch mass.
Number of trials	During the experiment, we conducted each launch twice to minimize the errors that occur during the experiment.	A much better conclusion could have been reached if each launch had been repeated 5 times to reduce the uncertainties and minimize the effect of errors that occur during the experiment.
Different rocket shapes	We only used one rocket during all launches. The shape that I used may not have been the most effective shape to reduce air resistance and achieve a bigger range.	To improve this, we could use different rockets: different in size, different placement of the fins, different nose cones. This could help me understand about what the most ideal shape of a rocket is.
Precision of equipment	The equipment/tools used didnt have the most ideal accuracy. For example, I used a scale with a precision of ± 1 g and an electric pump with a reading uncertainty of ± 0.05 l.	By using more precise equipment with a better accuracy could improve the experiment and the conclusion reached.

Table 9 Errors and improvements of the study