Determining Player Strategies for Penalty Kicks in Football

Nikhil Harish Lakshman

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In this paper, mixed strategies for the kicker and goalkeeper in a penalty kick in football are determined to truly tell if randomising strategy is important to maximise goal/save output. Probability tables are cited and combined with variables to form a combination. This combination is manipulated using partial derivatives to determine the variables. These variables ultimately give us the mixed strategies for the kicker and keeper. We find that both the kicker and keeper have a preference to the right side of the goal (from the goalkeeper's perspective) and the distribution is more even in the case of the kicker which indicates an advantage. Given the lack of research papers on this topic, this analysis helps add more content to this area. All in all this paper serves as an analysis to determine the mixed strategies for both the player and the keeper.

1 Introduction

Penalty kicks are an integral part of football. Games and trophies are won or lost on penalties. A penalty kick in football is awarded when a player is fouled in the opponent box or the ball hits the opponent player's hand as per the handball rules. A kick is taken by a penalty taker against the goalkeeper from 12 yards out. Being a favourable position to score from, the probability of scoring a penalty (expected goals or xG for short) from a penalty, according to Opta is 0.78 [1]. Opta is an analyst used to collect data for players and teams across the world. A penalty can be visualised as a zero sum game between the penalty taker and the goalkeeper. Each player has his/her own strategy and try to maximise their payoff. In the case of the attacker, the payoff is scoring the penalty and conversely the payoff of the keeper is saving the penalty.

Table 1 Goalkeeper

KICKER	Left	Middle	Right
Left	0,0	1,-1	1,-1
Middle	1,-1	0,0	1,-1
Right	1,-1	1,-1	0,0

Table 1 illustrates the payoffs when the kicker and the keeper choose a particular direction to shoot or dive respectively. Left refers to the left hand side of the goal from the kicker's perspective and so does the centre and right. The best payoff for a kicker is +1 and a payoff for a keeper would be 0. This is because the keeper doesn't gain any value for the team by saving the penalty: rather, he prevents the team from losing value.

This is a very basic payoff matrix and for the most part is inaccurate. It has been included to give an idea of a basic payoff matrix without factoring experimentation and real life skill. The underlying numbers are different. It isn't always the case that when the kicker and keeper go the same direction the ball is saved.

The main purpose of this paper is to determine the optimal mixed strategies for the kicker and the keeper in a penalty kick. There is a very small amount of existing literature on this particular topic. Moreover, penalties are an important factor in a football game as they can determine the outcome. As a mini game itself the penalty is interesting. For these reasons, doing research on this is important, More information on this topic can be found in section 3.1. In the case of this paper, the analysis is strictly mathematical and excludes psychological factors in the analysis. Other factors will be brought up towards the close of the paper as we further discuss the results. Four variables representing the probabilities of the kicker going left and right and the keeper going left and right will be used. The probabilities for the centre will be a combination of the variables. Many people would say the player would shoot towards the side opposite to his foot (if he's right footed, he'd shoot left and vice versa) as its naturally easier. However, this doesn't account for keeper tendencies to choose that particular direction as no direction is naturally easier for the keeper. The results of this analysis could produce a distribution that says the kicker may shoot in the opposite direction more often.

2 Methods

A table containing data on how often a shot is stopped given the direction in which the player shoots and the direction in which the keeper dives will be cited. The mixed strategy will be in the form of percentage values for 3 possible decisions for the player and keeper. Consider the following variables which come under the mixed strategy.

Note that the right and left sides are from the keeper's perspective

2.1 Kicker strategy:

Kick left=x
Kick right=y
Kick middle=1-x-y

2.2 Keeper strategy:

Dive left=p
Dive right=q
Stay middle=1-p-q

(The middle probabilities are 1-x-y and 1-p-q because these probabilities sum to 1 since the sum of the probabilities is 1.)

Let us take the payoff associated with these variables. Let us denote this by a function which has two parameters. F(a, b) is the function where "a" refers to the direction that the kicker chooses and "b" the direction which the keeper chooses.

Let us take the following variables:

$$G = F(\text{left}, \text{left}) \cdot p \cdot x + F(\text{left}, \text{centre}) \cdot x \cdot (1 - p - q) + F(\text{left}, \text{right}) \cdot x \cdot q$$

$$H = F(\text{centre}, \text{left}) \cdot (1-x-y) \cdot p + F(\text{centre}, \text{centre}) \cdot (1-x-y) \cdot (1 - p - q) + F(\text{centre}, \text{right}) \cdot (1 - x - y) \cdot q$$

$$I = F(\text{right}, \text{left}) \cdot p \cdot y + F(\text{right}, \text{centre}) \cdot (1 - p - q) \cdot y + F(\text{right}, \text{right}) \cdot q \cdot y$$

The resultant combination will be:

$$G + H + I$$

Let us set this equal to some value V. Therefore:

$$G + H + I = V$$

Now we'll take the partial derivative of this expression with respect to each of the 4 variables. Partial derivatives are necessary here to determine the critical points of the system of equations. They will give us the maxima. This can be done by setting the 4 resultant equations of the previous step equal to 0. The system of equations will then be solved and that will give the value of the 4 variables, determining the mixed strategy.

We use partial derivatives to determine the points of maximum for these probabilities. This will give us a point where the player can't improve his strategy upon deviating which establishes a Nash equilibrium, This means by using partial derivatives, the player is on a local maximum on his axis meaning the best possible strategy has been achieved.

3 Literature Review

When subject to a particular scenario, a player must take the most optimal decision. This decision is taken based off how the tactics functions. This study[2] determines how team tactics influence player decision making. One limitation is that the study doesn't account for the possibility of the player dribbling with the ball.

Space is one of the most essential elements in football. Creating and exploiting space is necessary for a style of play to work. Teams have tried to contrive tactics to more efficiently occupy space and force possession turnovers.[3] covers the use of Voronoi diagrams to analyse dominance of space and movements. Analytical geometry is used in the above to analyse the dominance of space. He concludes that the Voronoi diagrams are inadequate to communicate the topic at hand. It is due to the fact that they're used to compute areas for still objects and don't have parameters for moving objects. For measuring dominance in area using simple kinematics, standard Voronoi diagrams with their polygon-like shape must be replaced by diagrams that exhibit new properties for better results.

Passing approaches are a basic element in football and networks of passes are what structure a game and influence how a game plays out. This paper[4] shows that no significant relation between decentralization of a team and their results has been found. This disagrees with Grund's (2012) theories on how a decentralized passing structure is more efficient. It also concludes that eigenvector centrality is a good way to analyse football matches.

Many have tried to evaluate overall team performance through analysing passing structures. This paper[5] defines a way passing structures can be evaluated to showcase how well a team does. Graph theory was used for this analysis. The author concludes that the study of passing networks in this paper is not perfect. Given the lack of z axis coordinates, a distinction between aerial and ground balls could not be made. Another constraint is the size of the data. With a larger data set, a more advanced coordination evaluation matrix can be constructed. Another paper that evaluates team performance is[6] which defines a football team as a network, and they use their analysis to create predictions for the 2014 World Cup and the 2013/2014 Serie A. A correlation between team performance and network indicators is observed.

This research paper[7] concludes that machine learning approaches can be used to value passes and rank players based on their skill. It is seen that the pass valuation model can detect playing styles with teams from the same league being in the same cluster and the top teams in that league are seen in a cluster of their own. They also show that valuing players skill can estimate their market values.

3.1 Game theory focused papers

Game Theory is well studied in football. This is a paper[8] where several complex models are constructed, each in relation with the other, in order to explain the Game Theory behind penalty kicks. An important method in this is counterfactual regret minimisation. The author concludes that large areas within the goal were never aimed for regardless of the combination of velocity and standard deviation. Moreover, when introducing the factor of the velocity of the kick, shots towards the centre should be hit with a lower velocity. When introducing the factor of faking the shot, the player never fakes it and shoots to the centre. He concludes that a real life penalty kick scenario is hard to imagine and any model would have to undergo significant simplifications.

Similarly, the study in[9] introduces various probability notations and tests the nature of mixed strategy equilibria in penalties. They check for this consistency across 459 penalty kicks and see if the kickers and keepers use the Nash equilibrium mixed strategy. They conclude that It is important to consider heterogeneity when conducting game theory related experiments.

There is also a paper that examines whether football players do use mixed strategies[10]. Around 286 penalties were a part of the dataset. In this paper, the accuracy of mixed strategy Nash equilibrium is determined for penalty kicks. They determine that MSNE is the closest to actual predictions. They also conclude that the kicker should consciously randomize his strategy for a penalty kick. If the kicker is aware of the keeper's save success rate, determining his strategy will be a lot easier. Another paper that uses game theory for predictions is[11]. This paper uses Game Theory to predict matches given a set of variables that encompass player skill, experience and the field factor. It concludes that game theory, differential calculus and stochastic simulations can be combined to compute the result of a complex human event such as a match of football.

To add on,[12] builds on the work of[11]. He uses the concept of the natural side and opposite side as used in[11] and provides an alternate way of testing mixed strategy equilibria for penalty kicks.

3.2 Applications of these papers

This type of research is usually carried out by analysts at football clubs to find and implement new tactics that can beat the competition. As the game evolves, coaches have evolved along with it. For example, the concept of a full back or center back inverting into the midfield to provide a numeric advantage in the midfield and to form the box like structure that can bypass the opponent midfield was an innovation that became more and more popular[13]. Jurgen Klopp's Liverpool were on a bad run of form. However, when they switched to this new inverted full back tactic, they went unbeaten in the last 10 games of the season. Analysts keep trying to find new ways to use their players with players now having more than one role to play regardless of what position they are in. This has also changed the approach towards transfers in football; many teams are now opting to sign defenders who are comfortable on the ball to help with ball distribution and explosive full backs who can either overlap the wingers or invert as mentioned above. Hence, analysing football using data, math and computer science is becoming increasingly important.

Moreover, new metrics for quantifying elements of the game that didn't exist before are coming into play. For example, the concept of expected goals which was introduced in 2012 has become a game changing and insightful metric that is now a common and irreplaceable statistic in every football match[14]. Another example is expected threat which measures how prolific a player is based off what they do on the ball[15].

From the fan's perspective, a lot of interesting facts can be learnt from these stats. One can observe individual stats or the team's stats. Individual stats include a player's heat map, dribbles, successful passes, goals, assists, possession retained, possession won and other similar stats. When looking at a team, one can see the conversion rate (percentage of the number of goals score to the number of shots taken) of the team, passes made, shots taken, shots conceded, expected goals conceded, expected goals and other similar stats.

There is extensive work on passes in football. How passes can be classified, relations with scoring goals, passing approaches and passing networks have all been studied in detail. Through this work, it is now known how to predict games based off how a team passes the ball around. There are results which define a relation between a team's passing structure and their overall chance creation. It has also improved the classification of passes, with new ways to term passes coming into the equation.

Research outside of passing involves work on player decision making, individual roles in chance creation, analysis of the use of space in football and research based on positional data. The results of these papers include knowledge on how dominance of space facilitates a team's performance, how player's take a decision to maximize the expectancy of a particular objective and other models that determine the probability of goals scored given a set of factors or how efficient a team's chance creation is with respect to their players.

For my research, I will be relying on this paper[16] for probability tables (Probability of scoring and how often players shoot in a particular direction).

Determining the mixed strategy is quite interesting as it often is not like what we think the strategy is. It reveals an interesting distribution about how a player and a keeper choose their directions and determining this mixed strategy is important for analysts and teams as it tells them what the average penalty scenario looks like. Moreover, mixed strategy for penalty kicks is quite a common topic and hence the results of this paper will be meaningful.

4 Analysis

Let us take the two tables as mentioned from paper[16].

Table 2 Jump Direction

KICK DIRECTION	LEFT	CENTRE	RIGHT	TOTAL
LEFT	18.90%	0.30%	12.90%	32.20%
CENTRE	14.30%	3.50%	10.80%	28.70%
RIGHT	16.10%	2.40%	20.60%	39.20%
TOTAL	49.30%	6.30%	44.40%	100.00%

Table 2 provides a distribution of how often the kicker and the keeper choose a particular direction.

Table 3 Jump Direction

KICK DIRECTION	LEFT	CENTRE	RIGHT	TOTAL
LEFT	29.60%	0.00%	0.00%	17.40%
CENTRE	9.80%	60.00%	3.20%	13.40%
RIGHT	0.00%	0.00%	25.40%	13.40%
TOTAL	14.20%	33.30%	12.60%	14.70%

Table 3 represents the probabilities of stopping a shot when the kicker and keeper choose a particular direction.

The above data was formulated after 311 kicks, 3 Judges were appointed to determine the side to which the penalty was shot. 18 of these kicks were off target and excluded from the analysis. They eliminated kicks which they all disagreed upon which was 7 kicks.

For our analysis, F (a, b) is modelled as the function that gives the probability of scoring. This would mean F (a, b) would be the result of another function subtracted from 1. Let us call this G (a, b) which denotes the probability of saving when the kicker chooses direction “a” and the keeper chooses direction “b”. Therefore:

$$F(a,b) = 1–G(a,b)$$

Now that we can calculate our F (a, b) values, let us determine the equation. Using the combination from section 2 with the values from the above table we get:

$$G = (1-0.296)px + (1-0)x(1-p-q) + (1-0)qx$$

$$H = (1-0.098)p(1-x-y) + (1-0.6)(1-x-y)(1-p-q)e$$
$$+(1-0.032)q(1-x-y)$$

$$I = (1-0)py + (1-0)y(1-p-q) + (1-0.254)qy$$

$$G = 0.704px + x(1-p-q) + qx$$

$$H = 0.902p(1-x-y) + 0.4(1-x-y)(1-p-q)$$
$$+0.968q(1-x-y)$$

$$I = py + y(1-p-q) + 0.746qy$$

As established in Section 1.2,

$$V = G + H + I$$

Now, taking the partial derivatives with respect to each of the four variables:

$$\frac{\partial V}{\partial x} = 0.704p + 1 - p - q + q - 0.902p$$
$$-0.4 + 0.4p + 0.4q - 0.968q$$

$$\frac{\partial V}{\partial y} = -0.902p - 0.4 + 0.4p + 0.4q - 0.968q$$
$$+p + 1 - p - q + 0.746q$$

$$\frac{\partial V}{\partial p} = 0.704x - x + 0.902 - 0.902x - 0.902y$$
$$-0.4 + 0.4x + 0.4y + y - y$$

$$\frac{\partial V}{\partial q} = -x + x - 0.4 + 0.4x + 0.4y + 0.968$$
$$-0.968x - 0.968y - y + 0.746y$$

Further simplifying, we get:

$$\frac{\partial V}{\partial x} = 0.6 - 0.798p - 0.568q$$

$$\frac{\partial V}{\partial y} = 0.6 - 0.502p - 0.822q$$

$$\frac{\partial V}{\partial p} = 0.502 - 0.798x - 0.502y$$

$$\frac{\partial V}{\partial q} = 0.568 - 0.568x - 0.822y$$

Now, to determine the critical points, let us set all of these individual equations to equal to 0. We set this equal to 0 to determine the point of maximum. These are the resultant equations:

$$0.6 - 0.798p - 0.568q = 0$$

$$0.6 - 0.502p - 0.822q = 0$$

$$0.502 - 0.798x - 0.502y = 0$$

$$0.568 - 0.568x - 0.822y = 0$$

Solving the system of equations, the values of the variables are:

$$x = 0.34385$$

$$y = 0.4534$$

$$p = 0.41908$$

$$q = 0.47894$$

5 Results

Table 4 Kicker Mixed Strategy

Left	Centre	Right
34.39%	20.27%	45.34%

Table 5 Keeper Mixed Strategy

Left	Centre	Right
41.91%	10.19%	47.90%

The mixed strategies for both the kicker and keeper have been determined. Both the kicker and the keeper should shoot towards the keeper's right more often than not. This shows that the keeper has to deviate from his preferred strategy of jumping to the left (Upon comparing to Table 2). Both sides modify their strategy of shooting down the middle. The keeper is aware that the kicker may choose the centre and should increase his probability value of staying in the centre. The kicker is aware that the keeper has made this strategy change and reduces his probability value. It is clear that both players prefer to pick a side. The kicker's strategy, however, is more distributed.

This also serves as substantial evidence that the kicker has the advantage in a penalty kick. By staying in the middle, the keeper takes a big risk because if he concedes the penalty, it is directly detrimental to the team's game. By staying in the middle, the keeper has a higher chance of conceding given that the kicker is more likely to shoot towards a side. Hence, staying in the middle is a big risk. However, even if the kicker misses the penalty, it isn't directly detrimental to his team's game. That is why the keeper is likely to pick a side meaning that the kicker can kick it down the middle a lot of the time without having the kick being saved. The kicker, however, doesn't opt for a majorly centric approach. He distributes his strategy evenly. He gives a lot less priority to the left than the keeper thinks he does. Given that the mixed strategy distribution is more even and the dilemma is a lot less taxing on the kicker, he is at an advantage.

6 Discussion

We visualize the penalty kick as a mini game between two players: The kicker and the keeper. We devise a method to determine mixed strategies for each and compare it to their preferred sides. A conclusion that can be drawn is that shooting or saving down the middle is overlooked and the sides are preferred. While it is understandable that the goalkeeper risks a goal being scored if he stays in the middle, the kicker can use this to his advantage and focus on the centre more often than his current strategy. The mixed strategy of the kicker relies less on aiming centrally. There are a few limitations to this analysis. The analysis does not account for more zones. The goal is broadly classified into left, right and centre. There is scope to divide it further and provide a more insightful distribution. Moreover, another factor that can be accounted for is player skill. This analysis is general for all penalty kicks. It is independent of variables like the player's conversion rate or the keeper's save percentage. Incorporating this could lead to a case specific method to determine mixed strategies. Ball velocity could also be included in an analysis like this. If the ball is hit slower it is bound to be more accurate than a ball that is hit faster. Moreover, psychological factors shape the result of penalties due to the pressure factor. However, it is hard to account for how it impacts the values of probability here. In the future, more specific methods for analysis can be developed. Perhaps, a new combination can be developed and can be partially differentiated to more accurately determine the critical points. There is a lot of scope for future work on this topic.

7 Conclusion

We find that both the kicker and the keeper are more likely to go for the right side (from the keeper's perspective). The distribution is however more even in the case of the kicker as compared to the keeper. This suggests that the kicker is more open to mixing his strategy than the keeper showing that he has an advantage in this situation. All in all, this serves as a good summary of the math behind penalty odds and strategy.

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