Omni-directional Robots Based on the Mecanum Wheel

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This paper focuses on the motion control of an omnidirectional mobile robot. A control method based on the inverse input-output kinematic matrix is explained. As the geometry of the robot and the kinematics of how it functions have important impacts on the robot performance this paper considers these two aspects and shows how an omnidirectional robot function. The paper illustrates the inverse and direct solutions to the calculations of the classical mechanics of machinery.

Introduction

Recently, autonomous mobile robots have been attracting intensifying attention due to their extreme significance for daily human life in many uses that include mechanical and medical aspects. This type of movement categorizes the mobile robot into omnidirectional and nonholonomic. Omnidirectional mobile robots have enticed greater regard on account of manoeuvring ability; that is, they can modify the course of motion without needing to employ difficult steps compared with the conventional mobile robots. Although the mecanum wheeled bot is not alone in the field of omni mobile robots, it outperforms other omni robots like the legged robots in ordinary terrain due to its energy productivity and reduced complexity.

This presents several military applications and medical uses where turning the robot at every point is less than optimal. Quick and easy access of machines at every point is useful. Congested environments can be easily explored, places that are difficult to explore by mankind but need the agility of humans can be conducted by omni-bots. Space exploration, military scavenging and medical examination would all take a leap if the speed of this robots could be made faster with the same manoeuvring abilities. The discovery of advantages that these robots present has already been done by other researchers.[1,2] The development of mecanum wheels was first recorded in several papers by mechanical engineers[3,4,5,6] and then the investigations were conducted by the control engineers[7,8] to retrieve data on these special wheels. The development of the new mecanum wheel led to new several studies that further developed new research[9,10,11] on omni robots.

Omnidirectional wheels or what have become known as special wheels are very extensive for omni robots. They were created to perform active motions in the direction perpendicular to the motor and passive movements in the direction of the motor, providing the mobile robot superior adaptability in a congested environment. Since the axes of these rollers are perpendicular to the axis of the wheel, this omnidirectional wheel design is famous as the four-sided wheel. The use of this version of the omnidirectional wheels led to many issues in terms of the platform movement accuracy and stability due to the vibrations and slippage because of the unstable contact concerning the wheel and the ground. This paper further explores the basics of the mecanum wheel making it available to students with even colloquial understanding of mechanics.

This paper is divided into several sections which help to go over the mechanics in order or to gather the info that's required. The first section is this the introduction followed by how the wheel is designed and how the rollers attached to the wheels are placed. There is a thorough description of how the roller is constructed on a graph and at what angles it is attached to the wheel. The next section focuses on the movement of the robot because mecanum wheels are special wheels that are differently constructed, and their distinct angled movement needs to be explained precisely.

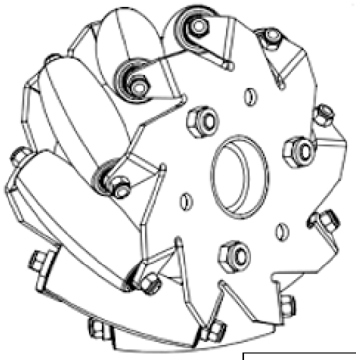
The last section where the kinematics are explained provides with essential description of classical physics where the whole concept of the wheel is explained from the basic forces that are acting on the bot to how different powers, torques etc., affect the forces in the x-y plane and how the angle or the twist on the wheel impacts forces.

Wheel design

Geometry of wheels

This section aims to explain the structure of the mecanum wheel, firstly starting from how the rollers would look on a graph on a x- y plane to finding out the shape of the rollers when cut on a 45-degree plane and how the final picture in figure 1 is aligned to be attached to the main robot. There are several rollers attached to the main wheel of the robot and it would be helpful to first understand the geometry of these rollers and how they are constructed. Shown below is a diagram of how these rollers are constructed.

Mecanum wheel[12]



Derivation of the roller has been explained below.

Figure 1

$\frac{1}{2}x^2 + y^2 = 4^2$

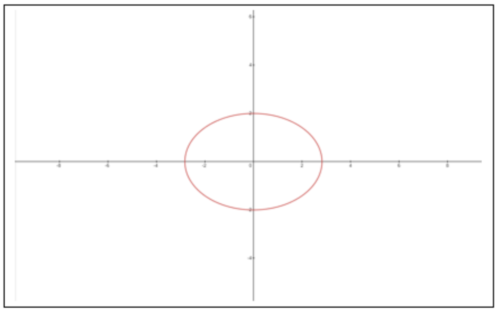


Figure 2

The rollers give diagonal movement to the robot as they are positioned at an angle. Understanding the shape and geometry of the roller would help to visualize it on a 2d plane (As shown above). It has the shape of an ellipse which is a circle extended on two opposite sides. The graph above represents the graph of the roller. This shape when revolved about the X-axis forms a 3d image of the multiple rollers that are being attached to the main wheel.

ii) Construction of wheels

Figure 3 visualizes the radius of the ellipse when placed on a 45-degree plane. This would help us find out the radius on all sides of the ellipse. As we can see in the Figure 4, we find out the diagonal values of the ellipse from its vertical and horizontal values. The line extended from the centre of the square to the centre of the edges has the value of R. So, by using the Pythagoras theorem we can find out the distance of the ellipse on the X-axis. Which in total comes out to be $2\sqrt{2}R \cdot \left(\sqrt{2^2+2^2} = \sqrt{8}\right)$ Figure 5 shows the values derived from the diagonal diagram to an elliptical structure.

The equation of the ellipse is $\frac{1}{2}x^2 + y^2 = r^2$. The value of R is the radius of the ellipse on the Y-axis Since the equation shows a half in front of the R the value of R on the X-axis in both directions would be $\sqrt{2}R$. The construction of the wheel is done by putting the main wheel in between. Figure 6 shows how a mecanum wheel would look when it has been disassembled and this would be the centre part or the main wheel which has been covered by two plates as shown in the first diagram. Both plates are placed to resemble the 45-degree angle of all rollers. These plates have multiple places for the ends of the rollers to get attached and to match the shape of the rollers they are twisted from the front into an angle securely attaching the centre and main wheel in such a way that it's fixed in a place, but the rollers can still freely move around

Movement using mecanum wheels

i) Mecanum wheel movement

The section aims to show how the robots movement works as per wheel movement. As mecanum wheels are special wheels

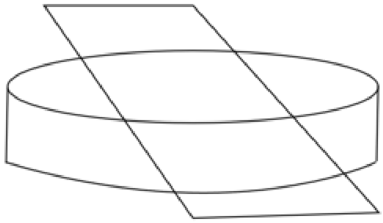


Figure 3

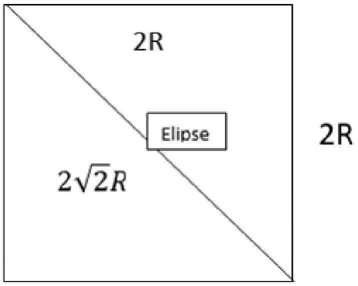


Figure 4

which don't move in the conventional manner the angular movement is given in depth explanation in the given section. This represents the geometrical model of the roller wheels and how the dimensions of the wheels would be.

Based on which direction the driver needs to move, we need to determine how fast to spin each mecanum wheel. Figure 7 shows the directions that each mecanum wheel exerts force. The easiest way to figure out how to spin the wheels in order to move in the desired direction is to make a table for each wheel. Full power forwards are 1, full power backwards is -1, and no power is 0. Another assumption to consider is that 2 will be the maximum output a battery can give to all its motors.

ii) Determining wheel speed based on direction

There are 4 motors in a mecanum wheeled robot two in the front and two in the back. If we assume that upwards is the forward direction and downwards is the backward direction this table can help understand the basic movements that can be performed using this omni-directional robot.

These values show the power input on every wheel for the basic movements of the omni-directional robot. To further understand how either forward or backward movement of all the wheels lead to different outputs of the robot we can see in Figure 8.

This demonstrates why mecanum wheels are so useful because they can guide the robot in any direction this helps them even turn around while staying at the same place. They can guide out of any place given their width and height. These makes them highly flexible and versatile. However, it would still be confusing to understand how these tyre movements lead to the overall output and the diagrams below can help.

iii) Force analysis for specific movements

A further explanation of forces by dissection of each force can be seen in Figure 9.

The main goal is to move leftwards and let's understand how we do that. The wheels 1 and 4 are moving in the forward direction. This helps see how the movement is in the top left direction. The other two wheels, 2 and 3, are moving backwards the overall movement goes in the bottom left direction. These diagonal forces can be further separated for movement on the X and Y-axis, horizontal and vertical direction. Let's first see 1 and 2, both have leftward movement in the left direction on the X-axis, but they have opposite movements in the Y direction, since we are giving them both the same power, they both cancel out leading to just a leftward movement. The same is the case for the tyres 2 and 3 where there is only movement leftwards as the Y directional forces cancel out. As shown above the overall movement in X direction is 4 times the leftward force of all wheels and there being two Y directional force upwards and two downwards there is 0 force. This leads to an overall force being

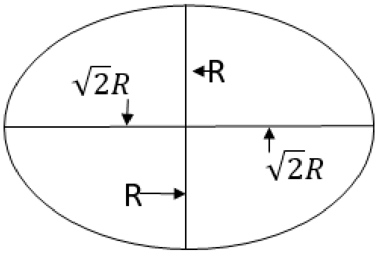


Figure 5

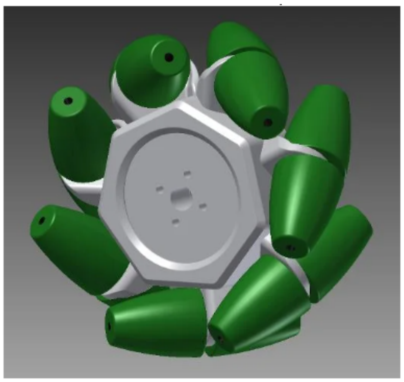


Figure 6[13]

produced only on the X-axis leftwards.

Reconstructing the diagram in the forward direction in Figure 10

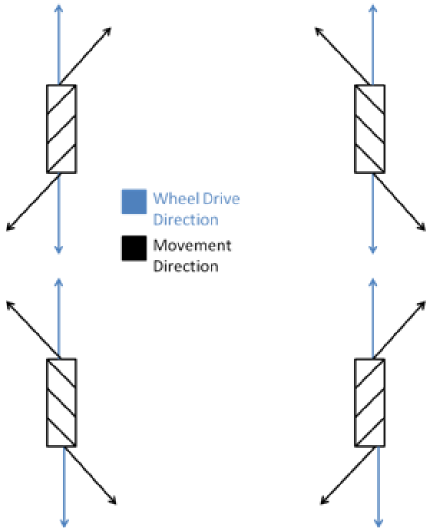
The main goal is to move forwards and let's understand how we do that. The wheels 1 and 4 are moving in the forward direction. This helps see how the movement is in the top left direction. The other two wheels, 2 and 3, are moving forwards and the overall movement goes in the top right direction. These diagonal forces can be further separated for movement on the X and Y-axis, horizontal and vertical direction. Let's first see 1 and 2, both have upwards movement in the forward direction on the Y-axis, but they have opposite movements in the X direction, since we are giving them both the same power, they both cancel out leading to just a forward movement. The same is the case for tires 2 and 3 where there are only movement forwards as the X directional forces cancel out. As shown above the overall movement in Y direction is 4 times the forward force of all wheels and there being two X directional force leftwards and two rightwards there is 0 force. This leads to an overall force being produced only on the Y-axis forwards.

Kinematics of the robot

With respect to the geometry the explanation of how the robot moves as per the given controls has already been explained but this section talks about how the physics behind this omni-bot works. It gives an in-depth explanation of classical mechanics showing how carious factors such as torque, power and forces lead up to the final movement of the omni bot.

The section first explains how velocity on each wheel is calculated. By use of the angular speed and the radius it helps us find velocity in a particular direction. Rearranging the equation in terms of torque establishes a relationship between torque, force and velocity. By using a diagram of the body of the robot a force vector diagram displaying movement in the x-y plane can be seen. Equations of force in x and y direction can be formed with the total torque. Putting the whole equation in terms of power allows us to find a matrix with power on each wheel as the subject. This helps us to know the travel of the robot when power is given. The matrix can be inversed to form an inverse-matrix that helps us know the power required to be input on each wheel when the force in each direction is known.

The equations and finding using classical physics ignore quantities that play a role practically. In theory, mass and friction acting on the wheel and the bot is ignored but this would



The Mecanum wheel has rollers positioned at a 45-degree angle, resulting in a diagonal movement. When the wheel would move forward or backwards the angular momentum would be at angles. For better understanding, an example can be taken so that when two forward wheels move forward, the 2 diagonally moving in opposite directions would lead to an overall forward direction. The battery would supply the motors a power in such a way that the main wheel either turns forwards or backwards, but all rollers are connected at angles leading to the diagonal movement of the robots which allows for the omni-movement. The movement of the robot in all directions according to the force applied can be presented in the diagram.

Figure 7[14]

	Top Left	**Top Right**	**Bottom Right**	**Bottom Left**
FORWARD	2-Jan	2-Jan	2-Jan	2-Jan
BACKWARD	-0.5	-0.5	-0.5	-0.5
FRONT-RIGHT	0	1	0	1
FRONT-LEFT	1	0	1	0
BACK-LEFT	0	-1	0	-1
BACK-RIGHT	-1	0	-1	0
Explanation = Speed represented by number(magnitude) and the sign represents direction of each motor. Battery can input maximum magnitude 2 in any direction.				

not result in the same output as the theoretical findings. A better way would be to simulate a 3d model and try on our own, but hardware testing needs to be done in the future we technologically developed models of these robots.

This section presents the kinematics of the mecanum robot, the angular velocities and velocity on each wheel is described below.

The velocity of the wheel when in contact with the ground can be calculated with this formula where we multiply the radius with the angular velocity. The radius is a straightforward value given depending on the wheel size and the angular velocity needs to be calculated. The equation above shows how the angular velocity is derived. This multiplied by the radius gives us velocity in a particular direction when in contact with the ground.

$$\vec{a} = \vec{r} \times \vec{a} \qquad (1)$$

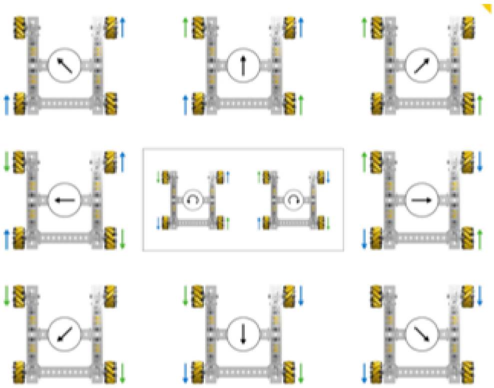
Equation 1 shows us how acceleration when in contact with the ground is the multiplication of the radius by angular acceleration. Newtons second law is force times mass = acceleration which can be written as

$$m\vec{a} = \sum \vec{F} \qquad (2)$$

This formula changes when we talk about the kinematics of rotations.

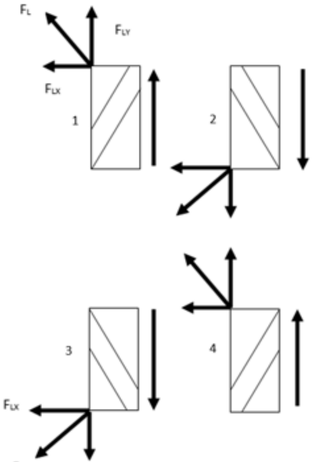
$$Ia = \sum T' \qquad (3)$$

Equation 3 represents the moment of inertia being multiplied by acceleration to find the total torque which is a type of force.



The blue and green arrows show movement on every wheel of the robot and the black arrow show the overall motion of the robot.

Figure 8[14]



MECANUM WHEEL MOVEMENT TO MOVE THE ROBOT LEFTWARDS

$F_{Total\,Y} = 0$

$F_{Total\,X} = 4F_{LX}$

$F_{Total} = F_{Total\,Y} + F_{Total\,X} = 4F_{LX}$

Figure 9

The torque is the force perpendicular to a radial arm extending from an axis, creating a torque T = force times the radius as shown above figure 12

The force shown in Figure 13 is the torque acting on the wheel, but when the acceleration is zero and there is constant speed It means the force is 0 because inertia times acceleration equals to torque but there must be some other force to make resultant force 0. The resultant force becomes zero because there is an opposing torque when the wheel is in contact with the ground which makes the robot move with a constant speed.

$$F_y = \frac{1}{R\sqrt{2}}(T_1 + T_2 + T_3 + T_4) \qquad (5)$$

$$F_y = \frac{1}{R\sqrt{2}}(-T_1 + T_2 + T_3 - T_4) \qquad (6)$$

$$T_z = \frac{F(l+w)}{2\sqrt{2}} \qquad (7)$$

The torque that had been explained above can be put into equations 5-7 to find out the resultant forces in the X and Y direction. The total torque can also be found by applying the equation. The length and width L and W are given in figure 15.

These equations are further substituted into a matrix to form a matrix that gives us the force on the x-y plane and the total torque when provided with the torque on each of the wheels shown in equation 8

$$\begin{bmatrix} F_x \\ F_y \\ T_z \end{bmatrix} = \frac{1}{R\sqrt{2}} \begin{bmatrix} -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ -\frac{(l+w)}{2\sqrt{2}} & \frac{(l+w)}{2\sqrt{2}} & -\frac{(l+w)}{2\sqrt{2}} & \frac{(l+w)}{2\sqrt{2}} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} \qquad (8)$$

There is a different power output on each of the wheels so firstly let us put the equation to find the power on each of the wheels

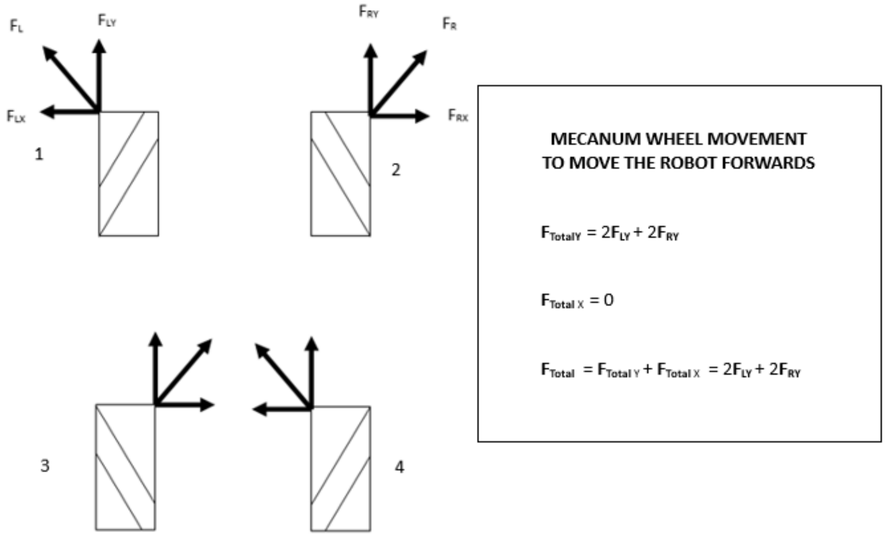


Figure 10

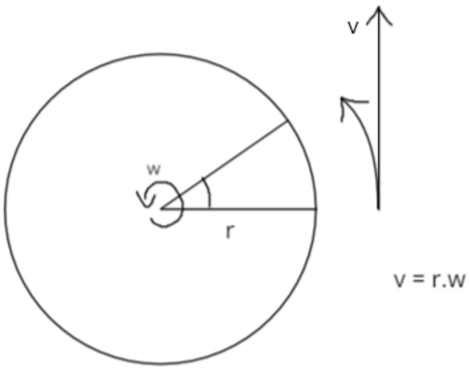


Figure 11

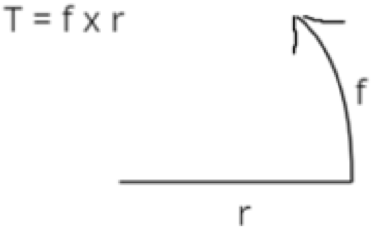


Figure 12

$$w_1 = \frac{1}{r} \times ((-l - w)(W_{\text{total}}) + V_x - V_y) \quad (9)$$

$$w_2 = \frac{1}{r} \times ((l + w)(W_{\text{total}}) + V_x + V_y) \quad (10)$$

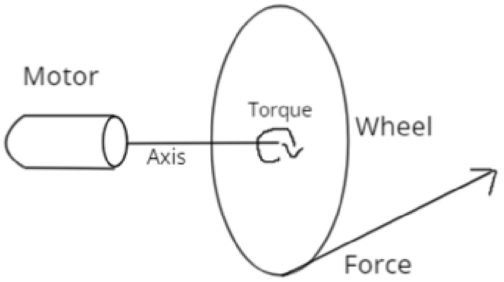


Figure 13

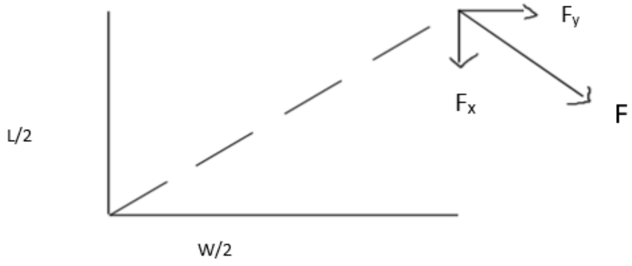


Figure 14

$$w_3 = \frac{1}{r} \times ((l+w)(W_{\text{total}}) + V_x - V_y) \tag{11}$$

$$w_4 = \frac{1}{r} \times (-l-w)(W_{\text{total}}) + V_x + V_y \tag{12}$$

To understand this concept lets break the equation one by one. Firstly, the 1/r that can be seen in front of each equation shows that to derive the power we have divide the equation by the radius of the main wheel. Then the angular velocity needs to be calculated by multiplying the length and the width of the robot by the total power. The L and W values are as follows.

The other two parts of all equations show the force in the x and y directions according to where the wheels have been placed, making it positive or negative. This graph can help understand that concept. The below diagram shows how the Velocities have been derived from the overall diagonal output.

There are 90-degree angles being made on each of the directions so using trigonometry we can see how the angle ϕ can help us find out what the force is.

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \frac{1}{r} \begin{bmatrix} -l-w & 1 & -1 \\ l+w & 1 & 1 \\ l+w & 1 & -1 \\ -l-w & 1 & 1 \end{bmatrix} \begin{bmatrix} W_{\text{total}} \\ V_x \\ V_y \end{bmatrix} \tag{13}$$

This is the matrix in equation 13 for inputting all these values to find out the power on each of the wheel, but we can reverse calculate and inverse the matrix to make an equation which can help us give the forces in both direction and the total power.

$$\begin{bmatrix} V_x \\ V_y \\ W_{\text{total}} \end{bmatrix} = \frac{1}{r} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ -\frac{1}{l+w} & \frac{1}{l+w} & -\frac{1}{l+w} & \frac{1}{l+w} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} \tag{14}$$

Equation 14 helps us input the power into each of our tires to find out the overall movement of the omnidirectional robot.

Conclusion

This paper demonstrates how omnidirectional robots function, including geometry, and how their kinematics function. This takes everything into greater detail to make it as easy as possible to understand the kinematics of these robots' function and how they are constructed for their specific use. This explains how the roller wheels and the robot is constructed to suit its purpose and how it has been aligned, it also explains the kinematics of the robot from the fundamentals. The provision of the basis of working of omnidirectional robots provides a basis for future papers to explore the topic. This would help make research

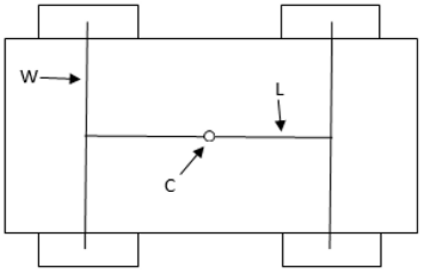


Figure 15

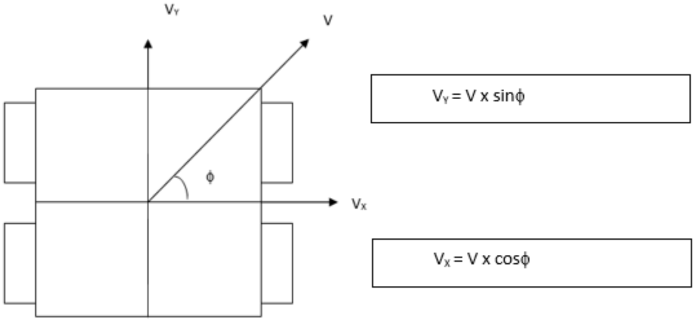


Figure 16

on future projects easier and launch huge scale robots into practical fields helping in large scale production in fields of defense, medicine and space exploration. High school students at all levels can get in-depth understanding of the kinematics as all explanation is divided into several segment for easier understanding.

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