Black Holes' Information Paradox and It's Complexity

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The information paradox in black holes arises from the clash between quantum mechanics and general relativity. It questions what happens to the information of particles that fall into a black hole. Resolving this paradox is essential for deepening our understanding of the fundamental laws that govern the cosmos and may lead to breakthroughs in unifying the laws of the micro (quantum) and macro (gravitational) worlds. In this work, we discuss the solutions that are proposed to the information paradox where information is released through a burst or it is still conserved inside remnants or event horizons. These approaches describe the thought process that developed in the past three decades to tackle this paradox. At the end we focus on the recent and most promising approach that includes the black hole's complexity and borrows quantum error correction concepts that are developed in quantum computing to understand the difference between perspective of an observer being inside and outside of the black hole.

Introduction

Black holes have always intrigued physicists due to their enigmatic nature. They are extremely dense objects that exerts massive gravitational force. At the center, the density converges infinity, a point known as the singularity. Apart from being a region from which not even light can escape, black holes follow the no hair theorem, which means they can be entirely characterized by their mass, charge, and angular momentum. Other characteristics are uniquely determined from these three parameters. "No hair" refers to the resemblance of a black hole to a bald head with few defining features. This already introduces a challenge, since it implies that the information of whatever fell into a black hole would not be able to be distinguished from outside the event horizon. Simply put, the result from an astronaut falling into a black hole would look the same as if it were instead an identical massive rock[1].

It was speculated that this information would be hidden behind the boundary where the velocity needed to escape exceeds the speed of light, also known as the event horizon of black hole. But in 1975, Stephen Hawking discovered that black holes emit the characteristic radiation which is known as the "Hawking radiation". One can think of Hawking radiation as a process that involves the creation of an entangled pair of particles near the horizon, out of which one particle escapes while the other is drawn into the black hole. The most significant implication of the Hawking radiation is that black holes gradually evaporate and eventually vanish. This means that either information is truly lost[2] in the black holes, or that information has been preserved by Hawking radiation by some mechanism.

In this work, first we establish the preliminary information that is required to understand properties of black holes. Then, the different approaches taken to solve the paradox are discussed and compared with each other. Approaches are ordered in a sense that each approach tries to overcome the short-comings of previous approaches. At the end we focus in more detail on the importance of complexity of black holes and why quantum error correction can help to understand black holes.

Black Hole Preliminaries

Unitarity

To understand the approaches to solve the information paradox there is a need to understand unitarity, pure state, mixed state and no hide theorem. Here we briefly describe these concepts before discussing information and entropy in black holes.

In quantum physics, unitarity refers to the condition in which the time evolution of a quantum state, as governed by the Schrödinger equation, is expressed mathematically through a unitary operator, a linear transformation that preserves all possibilities of a quantum state. This concept is typically regarded as an axiom or fundamental postulate within the framework of quantum mechanics. A unitary operator is defined as a bounded linear operator $U : H \to H$ on a Hilbert space H which has the following properties:

- It is bounded, which means it maps between finite spaces.
- U is surjective and preserves the inner product of the Hilbert space. In other words:

$$\langle Ux, Uy \rangle = \langle x, y \rangle$$

where $\langle , \rangle$ represents the inner product of two vectors. In quantum mechanics, the finite spaces that an operator maps between are the Hilbert space of the system being studied. This can be shown by the rotation operator as an example in Figure. In this example, two vectors $\vec{a}$ and $\vec{b}$ are shown. Consider that the Hilbert space is the plane in which the two vectors $\vec{a}$ and $\vec{b}$ lie. In this case, any rotation along the axis perpendicular to the plane will not keep the vectors $\vec{a}$ and $\vec{b}$ in the Hilbert space, and also, since the angle between the vectors does not change after rotation, the inner product stays the same. This can be seen in Figure 1.

Unitarity becomes evident through the no-hiding theorem[3], which asserts that if information is lost or destroyed it is actually situated somewhere else in the universe. This outcome arises from the fundamental principles of linearity and unitarity that are inherent in quantum mechanics, implying that information is preserved. This has far-reaching consequences, notably in addressing the enigma of the black hole information paradox, as well as in any process that appears to irreversibly erase information. Importantly, the no-hiding theorem maintains its integrity even in the presence of imperfections in the physical processes that seemingly obliterate the original information as black holes do.

In 2007, Samuel L. Braunstein and Arun K. Pati introduced the concept of the no-hiding theorem[4]. The theorem was experimentally verified using nuclear magnetic resonance devices, where a single quantum bit (qubit) goes through complete randomization and transitions from a pure state into a random mixed state. Interestingly, the information that appeared lost was retrieved from the ancilla qubits, which were introduced as an environment that interacts with the main qubit. This was made possible by applying appropriate local unitary transformations within the Hilbert space, resulting in the alignment with the principles of the no-hiding theorem. This experiment was a significant milestone, as it provided the first tangible demonstration of quantum information conservation. It shows that information about the qubit is not lost, it is just distributed between itself and its environment[5].

Quantum Pure and Mixed States

In the realm of quantum physics, a quantum state is a mathematical construct that encapsulates the information about a quantum system. The principles of quantum mechanics describe how a quantum state is formed, how it evolves over time, and how it can be measured. Quantum states can be divided into two categories, pure states and mixed states. In quantum physics, linear algebra is the most common way to formulate the theory. This involves identifying a given system with a Hilbert space that can be either finite- or infinite-dimensional. Pure states are represented by vectors with a norm of 1, and the set of all pure states corresponds to the unit sphere in the Hilbert space. This is because the unit sphere is defined as the set of all vectors with a norm of 1. And states that are not pure are mixed states. One important point to remember is that unitary operators can only evolve a pure state to another pure state, because unitary operators conserve the size of the vector[6].

Anti-de Sitter/Conformal Field

The anti-de Sitter/conformal field theory correspondence, often referred to as AdS/CFT or holographic duality, is a proposed connection between quantum field theory and a gravity theory in higher dimensions. This groundbreaking idea was introduced by Juan Maldacena in 1997[7] within the framework of string theory. Since its inception, this correspondence has undergone comprehensive validation and expansion. It functions as a connection between systems with strong and weak coupling properties, effectively mapping intricate, strongly coupled quantum field theory problems to more tractable, weakly coupled classical gravity problems. This duality offers a remarkable advantage: calculations that pose challenges within the field-theory domain become considerably more manageable within the gravity framework.

The AdS/CFT correspondence establishes a connection between high-energy particle physics and condensed-matter physics, bridging these realms with the domain of general relativity. It represents a novel manifestation of the unity that underlies diverse subfields of physics. One of the advantages of AdS/CFT is that it helps to study quantum gravity in the AdS space and generalize that to our universe[8].

Black Hole's Entropy

Knowing the mathematical concepts necessary, we can discuss the entropy of a black hole. As mentioned in the previous section, Hawking showed in his paper how black holes emit radiation[1]. In addition, Hawking showed that this radiation obeys a thermal spectrum with the characteristic temperature known as Hawking temperature T_{H} given by:

$$T_{\mathrm{H}} = \frac{\hbar\kappa}{2\pi k_{\mathrm{B}}} \quad (1) \tag{1}$$

In which κ is the surface gravity of black hole, $\hbar$ is the Planck constant divided by 2π, and k_{B} is the Boltzmann constant. Defining the temperature for black holes implies that it can have other thermodynamic properties and one can calculate its entropy. Before the discovery of Hawking radiation, Bekenstein proposed that the entropy of a black hole is directly proportional to its surface area. However, he was unable to determine the constant of proportionality from his equation, so the formula for the entropy of a black hole is known as the Bekenstein-Hawking formula:

$$S_{\mathrm{BH}} = \frac{k_{\mathrm{B}}A}{4l_{\mathrm{P}}^2} = \frac{k_{\mathrm{B}}c^3A}{4G_{\mathrm{N}}\hbar} \tag{2}$$

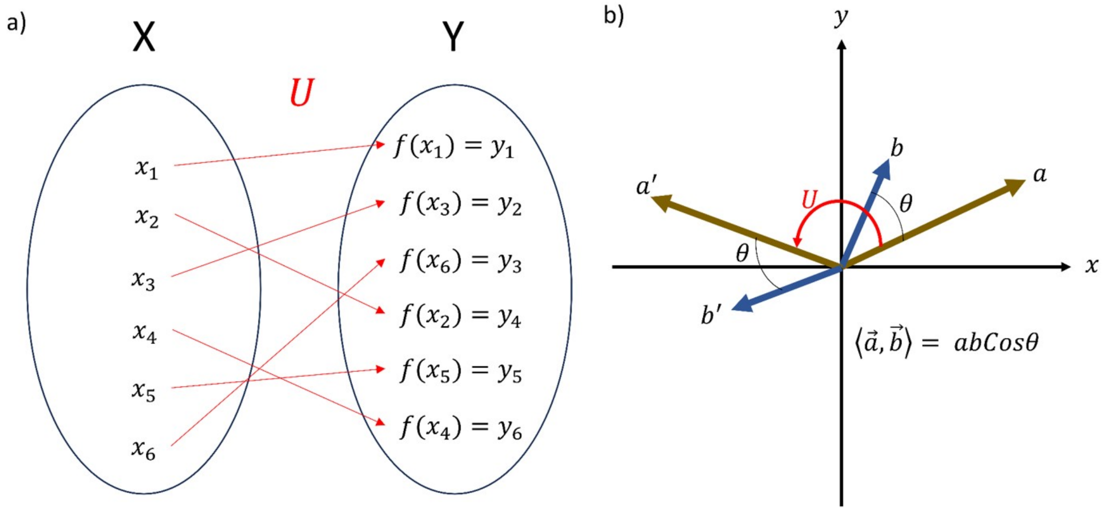

Fig. 1 a) Here U is an example of surjective function that maps between X and Y. b) Here rotation as a unitary operator is shown. In which, when applied to a and b vectors, it maps them to another point in the same Hilbert space and conserves the inner product of the two vectors.

Where $l_\text{P} = \sqrt{\frac{G_\text{N}\hbar}{c^3}}$ is the Planck length, and A is the area of the black hole.

As a black hole emits Hawking radiation and loses mass, the area of its event horizon decreases. Based on the previous equation, it may seem that the entropy would decrease over time. However, the total entropy of a black hole and its surroundings also includes a contribution from quantum fields outside the horizon. This total entropy is known as the ”generalized” entropy, which can be expressed as:

$$S = S_\text{BH} + S_\text{outside} \tag{3}$$

Where S_outside denotes the entropy of matter outside the horizon of the black hole[9]. In Hawking radiation, the particle that falls into the black hole is entangled with the particle that escapes. Therefore, there is entropy associated with this entanglement known as von Neumann entropy. The von Neumann entropy quantifies the amount of information present between the two particles. Knowing the density matrix of a quantum system, one can calculate the von Neumann entropy as follows[10]:

$$S_\text{vN} = -\text{tr}(\rho \ln \rho) \tag{4}$$

Where S_vN is the von Neumann entropy, tr describes taking the trace of the $\rho \ln \rho$ matrix, and ρ is the density matrix of the two-particle system. The density matrix contains all the information required to calculate the probabilities of the outcomes of any measurement performed on this system.

When matter undergoes gravitational collapse, a black hole is formed, which initially exists in a pure state. Therefore, the entropy of radiation is zero at this point. However, as the black hole begins to emit Hawking radiation, pairs of particles are released, and they become entangled with each other. If one only considers the outgoing particle, it is expected to observe that the outgoing particle is no longer in a pure state but in a mixed state. As a result, the entropy of radiation is no longer zero. However, as the black hole is evaporating, the area of its horizon is decreasing which means that Bekenstein-Hawking entropy should also decrease (Equation 2). This is problematic since this means that the von Neumann entropy becomes larger than thermodynamic entropy. However, this is not possible since the degrees of freedom of the black hole presented by thermodynamic entropy and entropy of the radiation cannot exceed it. Therefore, the entropy of radiation must begin decreasing at this point and continue to decrease to zero when the black hole completely evaporates. To summarize, it is expected that the entropy of Hawking radiation to follow the curve shown in Figure 2, called the Page curve, which refers to the work of Don Page in 1993 and brings the need to have this inflection point where entropy starts to be reduced and eventually reaches zero[11].

Different Approaches

The paradox remains without a clear resolution, yet numerous suggested solutions have emerged. These solutions span from the concept of encoded information within the emitted particles to the possibility of a baby universe, each attempting to address the paradox. The following approaches are organized in a way

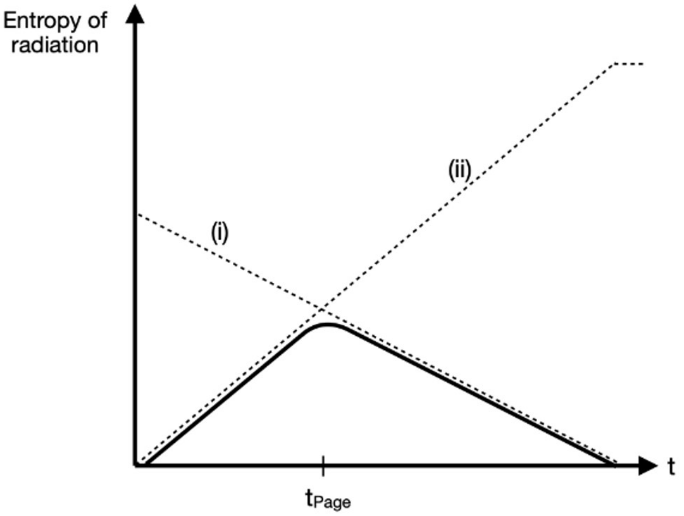

Fig. 2 (i) Represents the Bekenstein-Hawking entropy of the black hole (ii) represents the monotonic growth of the radiation entropy from Hawking's calculation until it fully evaporates. The solid line represents the Page curve, and t$_{Page}$ is the inflection point.

that each subsequent approach aims to address the limitations of the preceding ones.

Hawking Radiation Carries the Information

In this approach it is assumed information inside a black hole could be hidden in the radiation it emits, similar to how heat carries information in other systems. But there's a problem. The particles coming out of a black hole are entangled with each other. These particle pairs are created near the black hole, with one particle falling in and the other escaping. This entanglement should remain as the particles are getting far away from each other. One main issue that arises is that based on the no hair theorem, black holes should be fully described by their mass, charge, and their spin, but there is no clear method to describe this entanglement and information associated with it. An approach that follows the rules of physics suggests that a mechanism that is not discovered yet could erase the information in the particle that falls into the black hole. This would allow the escaping particle to contain all the information needed. This special situation is called the "unique state." When an object is in this unique state, it releases all its information through radiation and does not keep any for itself[12]. In this approach, the information is not lost but the entanglement is broken.

Information is Given Out in a Burst

To avoid breaking the entanglement, another approach is to consider that instead of encoding information in the radiation emitted by the black hole, information can be kept inside the black hole itself. This information would stay there until the black hole becomes very tiny (about the size of a Planck length, which is the smallest possible size according to theory), at which point it would be released all at once in a burst. This approach does not break the rules of causality or locality. Unlike the previous idea, this method avoids those problems because of the extremely small size of the black hole (around 1.616255×10^{35} meters).

However, while this approach respects the principles of physics, it introduces a challenge. Even though a Planck-size black hole has very little energy, it contains a lot of information. Initially, when the black hole is large, it holds a vast amount of information. Since this information does not come out as radiation but remains trapped inside the black hole, a Planck-size black hole ends up having low energy but a huge amount of information. Releasing such a large amount of information would not happen suddenly; it would require an incredibly slow process of emitting energy that would take longer than the entire lifespan of the universe. Also, thermodynamic objects with finite size and energy can contain only a certain limit of information given by the Bekenstein bound (Equation 5).

$$S \leq \frac{2\pi kRE}{\hbar c} \tag{5}$$

Where, S is the entropy of the object, k is Boltzmann's constant, R is the radius, $\hbar$ is Planck constant and c is the speed of light in vacuum. Since a Planck-sized black hole is small and has minimal energy, it would have to break the Bekenstein bound in order to hold an extremely large quantity of information[13].

Planck-Size Remnant Holds the Information

One way to address the shortcoming of the previous approach is by considering that the current theory of gravity, which combines aspects of quantum mechanics and gravity (the theory that underpins Hawking radiation), might not be valid for remnants that are as small as the Planck size. This is because at such a minuscule scale, these remnants could behave like particles. Quantum processes, like fluctuations at the quantum level, can have significant effects on particles. However, since we do not possess a comprehensive theory of quantum gravity, we can not accurately predict how a Planck-sized remnant would behave. Yet, one possibility is that these quantum processes could halt Hawking radiation and result in stable Planck-sized remnants that conserve information. This proposal adheres to the rules of the Bekenstein bound and causality, but it encounters a significant challenge. To store an arbitrarily large amount of information, these Planck-sized remnants would need to possess an infinite number of internal states due to their exceptionally small size. To tackle this hurdle, one suggestion is that a larger black hole could produce a multitude of these remnants, each with a finite number of states. This way, the total information could be spread across numerous remnants rather than requiring a single remnant to accommodate an infinite number of states[12].

Large-Size Remnants Holds the Information

Another possibility to consider is that the entropy of a black hole, known as the Bekenstein-Hawking entropy, is intricately linked to the size of the event horizon, which corresponds to the surface area. This entropy increases as the surface area grows, as indicated by Equation 2. In this scenario, we can assume that instead of a stable Planck-size remnant, we might have a considerably larger remnant, and its surface area would correspondingly increase along with the growth of entropy and information. However, there is a problem with this idea for a stable remnant to be larger than Planck-size, it would need to violate the principles of semiclassical gravity, specifically Hawking radiation. According to Hawking's theory, black holes are expected to radiate energy, which presents a dilemma for the concept of large, stable remnants that do not emit such radiation[12].

Event Horizon Holds the Information

Another solution is centered around the idea that a black hole's entropy is linked to the size of its event horizon rather than its volume. This concept is known as the holographic principle. It derives from the AdS/CFT conjecture, which proposes that information from three-dimensional objects can be encoded on a two-dimensional surface, similar to how a hologram stores three-dimensional information on a flat surface. According to this notion, the information of a black hole, being a three-dimensional object, is not lost within the black hole as its Hawking radiation particles fall in. Instead, it's duplicated onto the event horizon, which is a two-dimensional surface. This forms a stretched horizon. This configuration allows the information to exist in a way that does not violate the causality or the preservation of information, thus resolving the paradox.

Quantum mechanics, specifically the no-cloning theorem, states that information cannot be copied. However, the holographic principle appears to copy information onto the event horizon. This might seem like a violation of the no-cloning theorem, but it's not. The key is that both copies of the information cannot be observed by a single observer at the same time. It's argued that one copy can only be seen by an observer falling into the black hole, while the other is visible to an observer outside the black hole. This is termed the ”black hole complementarity.” Essentially, this principle explains that since the copies of information cannot be observed or shared between multiple observers, the no-cloning theorem is not actually breached, and the idea of unitary evolution remains intact.

The holographic duality sometimes also called the wormhole theory, in which it proposes that information sent through a black hole can emerge through another black hole linked by a wormhole, offering a shortcut in space-time. This concept, supported by physicists like Kanato Goto, involves a second surface within a black hole called the quantum extremal surface, connected to the outside world by multiple wormholes. Leonard Susskind and his team developed a practical proposal to simulate such wormholes by entangling quantum circuits and teleporting qubits between them[14], essentially making quantum particles mimic black hole behavior through quantum scrambling. While this approach has challenges, including the fragility of quantum entanglement and the need to perform calculations before decoherence, it provides a way to explore wormhole-like phenomena using quantum circuits, potentially offering insights into the mysteries of black holes and their connection to the AdS/CFT correspondence.

While the holographic principle and black hole complementarity successfully avoid disrupting causality or unitary evolution, they do come with their own limitations. One such drawback is that observables in the black hole also describe observables far from the black hole which implies a loss of locality in quantum gravity.

A Baby Universe Contains the Information

Another intriguing possibility arises where information conservation does not involve radiation or internal storage but rather entails transferring it to a separate universe known as a 'Baby Universe'[12]. Although this scenario is unlikely, it remains plausible owing to the presence of a singularity within a black hole. Given the singularity's infinitely dense nature, it allows for an extreme curvature of spacetime, potentially extending into another universe via a wormhole. In this conception, the information that descends into a black hole traverses the wormhole into a distinct universe, distinct from our own and characterized by its relatively diminutive size, hence the term 'Baby Universe'. This approach has the same shortcoming as the previous approach.

Computational Complexity

Next approach considers the complexity of black holes and has a history in learning the difference between inside and outside of the black hole. Since one cannot enter the blackhole, there is no direct information on what will be the observation of an astronaut falling in the blackhole versus his colleague outside of a blackhole observing him falling in the black hole. In 1993, Susskind and collaborators argued what an astronaut outside will tell is simply different from what an infalling astronaut will report[15]. An astronaut far away would witness their companion flattening onto the black hole's surface, which would ripple as it absorbed the trespasser. They would watch the information smear across the surface of the black hole and eventually turn into radiation, without ever disappearing inside. From the perspective of the falling astronaut , however, he safely enters the black hole, where both him and his information get trapped. Addressing these differences in observations introduced the concept of the observer's capacity to decode information from Hawking radiation. This decoding process is similar to solving a puzzle where its complexity is surging as more puzzle pieces are added. P. Hayden and D. Harlow[16] are the pioneers of this field and their computational complexity analysis shows it becomes more complex to decipher this data. It is important to define the complexity geometrically. In 2019, it was proposed that when there is more than one quantum extremal surface, the one that does not describe the entropy can be used to calculate the complexity of decoding the Hawking radiation[17]. It is proposed that multiple quantum extremal surfaces are the source of high complexity. This will identify the part of the geometry that Hawking's calculation reveals and the part of the geometry that Hawking's calculation cannot predict[18].

As it was mentioned quantum extremal surfaces are playing an important role. They are surfaces in the space-time that divide the space time in two regions. Hawking radiation has the information about everything inside the extremal surface, but does not have information about anything outside of the extremal surface. As the black hole emits more radiation, the quantum surface moves outwards and encapsulates more of the black hole. By the time that black hole completely evaporates the collected radiation should contain all the information about the black hole. There is a firm interpretation of the quantum extremal surface in AdS space which helps to extrapolate those understanding into a flat space as our universe which is an active field of research.

Knowing that the information inside the extremal surface can be decoded. The next question that comes is what would be this decoding process using quantum error correction. The currently developed algorithm contains two steps that one can imagine running on a quantum computer for converting the data coming out of the interior of the black hole[19]. It is argued that for any of the two-step processes, creating a complex semiclassical configuration with no parallel contribution from outside of the blackhole would essentially take eternity. Therefore there is no way to test the decoding process on a black hole directly. However, one could assume this decoding process works and try to see if contradictions rise from this assumption[20]. This is a continuing field of research and scientists are trying to apply this algorithm on different theories to find their caveats.

Table 1 Summary of the approaches to information paradox and their shortcomings

Approach to Information Paradox	**Approach's Shortcoming**
Information is encoded in radiation	The entanglement between the particle pair needs to be broken so the particle outside of the black hole will contain the whole information.
Information is given out in a burst	The large amount of information in the Planck-sized black hole cannot be released suddenly as it bursts and needs to be done slowly.
Planck-size remnant holds the information	Size of the remnant is too small to contain all the information. If the black hole is large, it may end with more than one remnant.
Large size remnant holds the information	For remnants larger than Planck size to be stable, they will violate the principles of semiclassical gravity.
Information being held in event horizon	Observables in the black hole also describe observables far from the black hole, which implies a loss of locality in quantum gravity.
A Baby universe contains the information	It has the same principle of information being held in the event horizon with the same shortcomings and it is a less likely scenario.
Computational Complexity of black holes	There is no direct method to test this theory.

Conclusion

The investigation into the information paradox within black holes is a critical pursuit at the intersection of quantum mechanics and general relativity, where the fate of information that falls into a black hole remains uncertain. Despite ongoing efforts, no conclusive answer has been reached. A comprehensive solution to the information paradox would describe how exactly the black information comes out. Therefore, an observer that knows the solution should be able to decode the information that is released such as information about the star that collapsed and became the black hole. This work presented the approaches that were done to solve this paradox in order to help establish a thinking path on how the resolution to this paradox is initiated and evolving. Finally we focused on the most recent approach that incorporates complexity and quantum error correction and mentioned that this is the only approach by now that provides an algorithm to compare the perspective from inside the black hole to the outside of the black hole. However, still there is no direct way to examine this approach. Therefore, the current subject of the research is to apply this approach to gravitational theories in space with different geometry and check if there are inconsistencies.

References

1 C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation*, Retrieved 24.

2 S. W. Hawking, *Physical Review D*, **13**, 191,.

3 S. L. Braunstein and A. K. Pati, *Physical Review Letters*, **98**, 080502.

4 A. K. and A. Kumar, *Physical Review Letters*, **106**, 080401.

5 L. Zyga, *Phys.org*.

6 C. Cohen-Tannoudji, B. Diu and F. Laloë, *Quantum Mechanics*, Wiley, p. 231–235.

7 J. Maldacena, *Advances in Theoretical and Mathematical Physics*, **2**, 231–252.

8 O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, *Journal of High Energy Physics*, **10**, 091.

9 A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian and A. Tajdini, *Reviews of modern physics*, **93**, year.

10 M. Nielsen and ILChuang, *Quantum computation and quantum information*, Cambridge University Press, 10th edn.

11 D. N. Page, *Physical Review Letters*, **71**, 3743–3746.

12 J. Preskill, P. Black Holes, W. Membranes and S. Superstrings, *Kalara and DVNanopoulos, eds.(World Scientific*.

13 S. Giddings, *The black hole information paradox*, arXiv preprint hep-th/9508151.

14 A. Brown, H. Gharibyan, S. Leichenauer, H. Lin, S. Nezami, G. Salton, L. Susskind, B. Swingle and M. Walter, *Quantum Gravity in the Lab: Teleportation by Size and Traversable Wormholes*, `https://doi.org/10.1103/PRXQuantum.4.010320`.

15 L. Susskind, *Physical Review Letters*, **71**, 2367–2368.

16 P. Hayden and G. Penington, *Commun. Math. Phys*, **374**, 369–432.

17 A. Almheiri, N. Engelhardt, D. Marolf and H. Maxfield, *Journal of High Energy Physics*, **2019**, year.

18 P. Hayden and G. Penington, *Journal of High Energy Physics*, **2019**, year.

19 C. Akers, N. Engelhardt, D. Harlow, G. Penington and S. Vardhan, *The black hole interior from non-isometric codes and complexity*, `http://arxiv.org/abs/2207.06536`.

20 A. Kar, *Journal of High Energy Physics*, **2023**, year.